Modern Educational Technologies in a Fractal Approach Implementation in the Math Lessons (on the Example of Learning a Probability-Statistical Line Elements)

Sergey Victorovich Shcherbatykh1,* and Anna Yuryevna Polyakova2

1Vice-Rector for Academic Affairs, Bunin Yelets State University, Yelets, Russian Federation
2Bunin Yelets State University, Yelets, Russian Federation; the teacher of mathematics, Municipal State-Owned Educational Institution “Secondary School No. 3 named after O.A. Morozov”, Yefremov, Russian Federation

Abstract: The article aims to reveal the didactic potential of modern educational technologies used within the framework of the fractal approach in teaching stochastics to learners, to show the effectiveness of fractal approach technologies in practice experimentally. In the course of the scientific research, the authors employed scientific analysis of literary sources on philosophical and methodological problems associated with the introduction of a fractal approach in teaching and informatization of education; systematization and generalization of the principles of fractal pedagogy; study, analysis, and concretization of advanced pedagogical experience in the use of modern educational technologies in the educational process; observation and analysis of the results of educational activities of seventh graders; and pedagogical experiment. This research allowed for identifying a group of modern educational technologies in the implementation of the fractal approach in mathematics lessons and identifying their didactic potential and possibilities of using, which is reflected in Table 1 of the main text of this publication. At the same time, it was found that the technologies of the fractal approach in teaching are quite useful: the experimental group received the best result.

Keywords: Education, stochastics, technology, fractal approach, learners.

INTRODUCTION

Thanks to the revealed achievements of synergetics, a wide range of opportunities have opened up for building new models of teaching the exact sciences. Mathematics, namely, its section - statistical probability line, plays a notable role in education. Learning models are developed based on various approaches, one of which is fractal, building useful feedback in a dyad that opens up a dialogue space for communication between the schoolchild and the teacher. Thanks to such communication, a new culture is being formed – a joint activity culture that preserves the educational process subjects' uniqueness and prepares its participants for understanding and partnership.

Situations in which an object cannot be represented using the classical model due to non-linearity and non-determinism of data entail using a fractal approach. Because of such situations, it becomes necessary to develop a model of chaotic development, when the object of study can develop in several ways. Besides, through the fractal approach, the ability of dynamic systems to self-organize can be considered. Simultaneously, human beings and human society are the most critical factors affecting establishing such systems.

As is known, the fractal approach to teaching various disciplines contains the idea of a quantitative and qualitative assessment of the structures of educational elements. Simultaneously, educational elements can possess a complex hierarchical organization of any dimension (Dvoryatkina, 2015). Organizing the information flow with the establishment of various internal and external relations and using modern educational technologies are the problems that the fractal approach is focused on solving.

Educational models and technologies used within the framework of fractal pedagogy's conceptual ideas and the fractal approach to learning are incredibly useful and have a suitable development vector. The use of these technologies in their work allows the teacher to develop and realize society's most professional competencies.

Besides, let us say that the new paradigm of pedagogical science promotes the unification of structures that develop a student's specific properties and personal qualities in one sense chain. Information about these properties' features can be found in regulatory documents governing the current conditions' educational process. An example is the National Doctrine of Education in the Russian Federation for the
period up to 2025 (Specialized Educational Portal “Innovations in Education”, n.d.), proclaiming the primary education goal, firstly, the person self-realization and self-education skills development, and secondly, holistic and modern scientific worldview formation in learners.

The individual self-realization and self-education within the framework of the conceptual ideas of the fractal approach can occur through using the following modern educational technologies: humanitarian, fractal and neural networks, gaming, health-saving, reflective-evaluative, case-based technology, collaborative learning technology, web-quest technology, personal achievement assessment technology, holographic vitagenic learning technology with projection method. This manuscript contains a description of the possibilities and results of applying the mentioned technologies when teaching probabilistic-statistical lines to learners - the stochastic one.

It should be noted that the proposed educational technologies list in the fractal approach vision provides an opportunity to qualitatively change not only the taught math courses content and structure but also to increase the technological training effectiveness by combining educational and digital content, new forms and teaching methods.

According to the authors’ point of view, the guaranteed learning outcome, which consists of the depth of students’ understanding of mathematical material and quantitative indicators, is directly related to the teacher's personality's harmonious development. The use of educational technologies considered by us in the context of the fractal approach will contribute to the teacher’s quick adaptation to changing living conditions. The teacher will develop the ability to think critically and solve emerging problems in education and the surrounding reality. The provision of the ability to work competently with information and independence in intelligence development should be added to the previous paragraphs.

The article's materials can be useful for mathematics teachers, students, and graduate students when using modern educational technologies in practice and work based on a fractal approach to teaching. The article's materials provide the author's definition of a “fractal approach in teaching,” containing an updated list of educational technologies used in implementing the fractal approach.

LITERATURE REVIEW

Before exploring the potential of new educational technologies in studying the probabilistic-statistical line in the context of the fractal approach, it is crucial to know how the "fractal approach" concept evolved. This concept is related to the “fractal pedagogy” concept. The latter is used in a wide range of scientific areas: computer science, economics, mathematics, finance, biology, energy, radio engineering, pedagogy, and radar-location. Most of the phenomena and objects of the surrounding world, time, and space possess fractal structures. That is why it is worth pointing to the origins of fractal pedagogy, which turn to the definition of the concept of “fractal.”

It is worth noting that at the time of ancient Greek philosophy, albeit in an implicit form, there was an idea about fractals. Thus, one of the works of antiquity contains the idea of the cyclical nature of history. A modern interpretation would define this fact as the revealed fractal structure of the era.

According to Mandelbrot (2005, 2009) and Vojciechovich (2001-2004), fractals are found in various cultures and can embody themselves in future generations. Besides, fractals reproduce all life in natural phenomena, making it similar to its image. Thus, a fractal is a self-similar set of non-integral dimension (Meteshkin and Meteshkin, 2010).

A human being also faces the properties of fractal formations. These are found in all spheres of human existence. A non-trivial structure can be observed at all levels and facets of each member of human society's development.

Thus, the authors can affirm that the modern person is a fractal capable of self-organization and transformation of the world. The fractal structure in the activity context can be traced to the following chain: “a person producing tools” - “a playing person” - “a person of high technologies in the field of production, management, and education” - “a person of high humanitarian technologies” - “economic person” - “creative person” - “healthy person” (Madzhuga and Sinitsina, 2016). A derivative fractal from this chain is a person of high humanitarian technology and the field of activity in which this person is involved. Such a society member tends to develop and is a multi-level, multi-deterministic, hierarchical, and multi-dimensional structure, unique.
The activity of a teacher and a mentor is humanitarian. Interpersonal interactions in the educational environment can be seen as the result of fractal interactions (Madzhuga and Sinitsina, 2016). Interpersonal interactions in the educational environment and pedagogy itself, as a subject, not only engaged in the study of education and mentoring, meet fractal properties.

The fractal pedagogy is a combination of fundamental scientific principles, ideas, and terms of the modern pedagogical science, which include the following similar structures: training, education and self-education, upbringing and self-upbringing, and development and self-development. These function in this system under the principle of resonance. There is a constructive and creative dominant capable of transforming a person's resource potential in the process of creative activity (Madzhuga, Sinicina, and Filipenko, 2015).

Studying the fractal pedagogy objects are the specific laws of education, development, and upbringing determined during teaching stochastics to learners. The subject of studying the fractal pedagogy is considering its self-similar structures through the prism of the fractal approach.

Fractal pedagogy in the study of the probabilistic-statistical line allows for solving the following tasks of the theoretical and empirical levels:

1. Considering the features of the schoolchild's assimilation of sociocultural experience in the educational process;
2. Revealing the essence of self-organization and non-linearity of the fractal system of education in the process of teaching mathematics;
3. Substantiating theoretically the distinctive features of the upbringing and development of a schoolchild in the process of teaching a probabilistic-statistical line, based on the provisions of the fractal approach;
4. Studying the concept of the fractal essence of the schoolchild;
5. Identifying the general and particular patterns of formation and development of the personality of each schoolchild based on the principles of the fractal approach;
6. Substantiating theoretically the teacher's use of modern methods, forms, and means, as well as didactic opportunities in stochastic lessons;
7. Designing the educational process in the study of stochastic material based on the laws of the method of self-organization and fractal theory;
8. Identifying the methods and techniques for the development of educational material that contribute to the formation of probabilistic thinking style in learners;
9. Identifying the main ways of development of learners, thanks to which their self-development and independent activity is realized in stochastic lessons and when attending electives and clubs;
10. Developing and proposing the criteria to determine the fundamental potentials of teaching and raising children during the school course's probabilistic-statistical line in mathematics (these potentials include creative, motivational, intellectual, creative, etc.).

The above-listed tasks entail observing the main twelve principles of fractal pedagogy (Madzhuga and Sinitsina, 2016).

The principle of non-linearity in teaching stochastics to learners implies unpredictability of the result, which cannot be predicted due to the training course content's mismatch with the system of learners' developed competencies. In this case, it is infrequent that the learning process of the elements of statistics, combinatorics, and probability theory and the learning outcomes obtained are identical to the teacher's intent. Solving educational problems requires creativity and a probabilistic style of thinking among participants of the educational process.

The second principle of congruency claims a direct relationship between the teacher and the student: students' inspiration to learn the basics of stochastics will be manifested mainly if the teacher is passionate about the course.

The third principle of fractal harmony indicates each schoolchild's integral development as a fractal system, the elements of which are closely interconnected and interdependent with each other.

The fourth principle of openness makes any pedagogical system open to teachers and learners...
from other education levels and representatives of various professions.

The fifth principle of hierarchy implies creating such an educational program in which the subject minimum of discipline for each level of studying the stochastic line is already defined.

The consequence of the sixth principle of mutual dominance and trust in human nature is respect for each schoolchild’s and the teacher’s personality and recognition of each educational process subject's personality as unique and peculiar.

The seventh principle of resonant interaction is expressed in the teacher's ability to convey the sociocultural experience to learners, while at the same time creating a positive internal mood in them.

The eighth principle of a holographic projection builds an effective educational process based on states with three projections having center-directed vectors. The projections include vitagenic, didactic, and construction (Belkin, 1998, 2000). Vitagen projection is the information requested by the teacher in preparing for the presentation of new material on one of the topics of statistics, combinatorics, and probability theory. The guiding vector is the following chain: schoolchild - knowledge - teacher. The didactic projection is represented by scientific information from the teacher and determined by the vector: teacher - knowledge - teaching. The constructive projection transfers information from any additional information source and creates a holistic holographic picture of knowledge.

The ninth principle of the optimization of reflexive interaction is carrying out reflexive support and reflexive accompaniment of learners at the group and individual teaching stochastics.

The tenth principle of holism considers the schoolchild a component of the “personality - environment - educational environment” system. The psycho-emotional component and the body’s bi-morphological structure must be inscribed in the person's social being.

The eleventh principle of sameness ensures the internalization and assimilation of learners' social and cultural experiences in a creative and universal form.

The twelfth principle of hologram shows the mutual reflection of rational and irrational methods of cognition.

The indicated fractal pedagogy principles contribute to learners’ best subject-oriented education in the probabilistic-statistical line; describe the possibilities of developing self-organization skills, self-development, self-education, and the possibilities of creating trusting relations between teacher and learners.

The indicated fractal pedagogy principles contribute to the best subject-oriented learners' education in the probabilistic-statistical line; describe the possibilities of developing self-organization, self-development, and self-education skills, creating trusting relations between teacher and learners as part of a fractal approach to learning.

The authors understand the fractal approach in teaching stochastics as a set of methods for organizing the information flow, based on the principles of fractal pedagogy, with the establishment of various types of internal and external relations of the fractal system, as well as using modern educational technologies in the educational process.

MATERIALS AND METHODS

The authors used the following methods: scientific analysis of literary sources on philosophical, methodological, and psychological-pedagogical problems associated with the introduction of a fractal approach in teaching and informatization of education; analysis of scientific literature on the methods of teaching mathematics; systematization and generalization of the principles of fractal pedagogy; study, analysis, and concretization of advanced pedagogical experience in the use of modern educational technologies in the educational process; observation and analysis of the results of educational activities of seventh-graders; design, development, and use of teaching materials in the field of teaching the probabilistic-statistical line; a pedagogical experiment, which consists in testing some technologies of the fractal approach when studying the topic “Statistical characteristics.”

The theoretical and methodological base of the study was made up of:

- Theory of fractal approach in teaching (Byshimaeva and Razova, 2017; Dvoryatinka and Shcherbatykh, 2020; Dvoryatinka and Kuznetsova, 2016; Shcherbatykh and Polyakova, 2020; Travkina, 2018);

- Research on the theory and methodology of teaching the elements of the probabilistic-
<table>
<thead>
<tr>
<th>S/N</th>
<th>Name of the educational technology</th>
<th>Definition</th>
<th>Features of using the technology in teaching stochastics</th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>Humanitarian</td>
<td>Humanitarian technologies are technologies aimed at implementing programs of education, mentoring, and spiritual development of the individual. These technologies are based on the integrated use of knowledge about the schoolchild and its spiritual culture.</td>
<td>The use of these technologies gives the teacher high creative potential and develops the ability to see the educational problem from non-trivial perspectives. Humanitarian technologies put the teacher’s organizational skills and the ability to act here and now, that is, in real-time, to the forefront.</td>
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<tr>
<td>2.</td>
<td>Fractal and neural network-based</td>
<td>The fractal method of analytical diagnostics, prognosis, and correction of the human condition is the technology, whose main task is the self-expression of a person (a schoolchild or a teacher) through a drawing made according to specific requirements (Poluyakhtova and Komov, 2002). Neural network technologies are presented by a complex of information technologies utilizing software- or hardware-implemented systems built on the human nervous system’s organization and functioning.</td>
<td>When teaching stochastics, this technology will allow for understanding the essence of the educational process and adjusting it naturally if necessary. The fractal method devotes much attention to the spiritual component of a schoolchild, thanks to which the schoolchild’s creative abilities are manifested. These technologies allow for the optimal individualization of stochastic education and self-realization of each schoolchild’s personality.</td>
</tr>
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<td>3.</td>
<td>Health-saving</td>
<td>Health-saving technologies are those representing an integrated system of corrective and preventive, educational, and health-improving measures implemented in the course of interaction between the subjects of the educational process (of the schoolchild and the teacher) (Serdyukova, 2018)</td>
<td>Technologies actively influence the state of development of the schoolchild’s body, the formation of the schoolchild’s behavior style, and their habits.</td>
</tr>
<tr>
<td>4.</td>
<td>Gaming</td>
<td>Gaming technologies are represented by various psychological and pedagogical methods, ways, and educational and mentoring tools (Vaganova, Smirnova, and Mokrov, 2019).</td>
<td>The use of gaming technologies meets the requirements of self-expression of the schoolchild’s personality, knowledge of own personality, and other educational process participants to find ease in communication and the most effective assimilation of educational material.</td>
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<td>5.</td>
<td>Vitagenic education with a holographic projection method</td>
<td>Vitagenic education with a holographic projection method is the education carried out by updating the schoolchild’s life experience (Belkin, 1997). A system of means and methods of volumetric multi-dimensional supply of the studied material is used (Brusnitsyna, 2005).</td>
<td>This educational technology helps form a social image of the schoolchild’s personality, in discovering the resources hidden in the child’s subconscious. The teacher becomes an inspiration for the comprehension of new knowledge.</td>
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<td>6.</td>
<td>Reflexive-estimative</td>
<td>Reflexive-estimative technologies are those whose purpose is creating conditions for conscious learning of the probability-statistical line by the learners and forming and developing learners’ ability to learn by mastering the control and evaluation actions.</td>
<td>The reflexive-estimative analysis of methods for solving stochastic problems and the development of universal educational actions in the study of the discipline and their correction are essential for reflexive-estimative educational technologies. Because of the application of these technologies, learners have the ability for self-regulation and self-education in general.</td>
</tr>
<tr>
<td>7.</td>
<td>Portfolio technology</td>
<td>Portfolio technology is the one to select, organize, and analyze information on a selected topic, work with various sources of information.</td>
<td>When teaching stochastics, portfolio technology can be used to demonstrate, analyze, and evaluate educational outcomes. Besides, with the help of this technology, self-consciousness develops, and the levels of self-awareness and self-esteem of educational activity among learners increase.</td>
</tr>
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## Table 1 (Continued)

<table>
<thead>
<tr>
<th>S/N</th>
<th>Name of the educational technology</th>
<th>Definition</th>
<th>Features of using the technology in teaching stochastics</th>
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<tbody>
<tr>
<td>8.</td>
<td>Education technology in collaboration</td>
<td>Education technology in collaboration is represented by joint or distributed education with the production of new knowledge.</td>
<td>This technology helps learners develop teamwork skills. Thanks to education in collaboration, educational problems are solved based on the resources of mutual experience and achievements in the probabilistic-statistical line of the school course in mathematics.</td>
</tr>
<tr>
<td>9.</td>
<td>Web-quest technology</td>
<td>Web-quest technology is an interactive form of learners' project activity, aimed at developing analytical and creative thinking and requiring a high level of pedagogical competencies from the teacher who uses this technology (Rogacheva and Shcherbatykh, 2014).</td>
<td>Web-quest technology encourages research and probabilistic thinking among learners, promotes independence, critical cooperation, and broadens a schoolchild's horizons.</td>
</tr>
<tr>
<td>10.</td>
<td>Case technology</td>
<td>Case technology is a technology for analyzing specific situations, a &quot;special case&quot; (Andriyenko, 2019).</td>
<td>Case technology has high potential training opportunities, is practical, and allows learners to form a wide range of professionally essential qualities among learners.</td>
</tr>
</tbody>
</table>

Statistical line (Shcherbatykh and Rogacheva, 2017; Shcherbatykh and Lykova, 2017; Polyakova and Shirshova, 2016; Shcherbatykh and Polyakova, 2018);

- Scientific work on the introduction of new educational technologies in the educational process (Zahro, 2019; Zimnyakova, Larin, and Larina, 2019; Ergashev, 2018; Yakubova and Parpieva, 2019);

- Study of the methodological features of the use of information and communication technologies in teaching mathematics (Gerbekov, Kubekova, and Chankayeva, 2016; Fasakhova, 2017; Zhumanova, 2020; Vasilyeva, 2016; Rogacheva, 2016a);

- Issues concerning forming students' stochastic culture through modern educational technologies (Rogacheva, 2016b; Polyakova, Shcherbatykh, and Chernousova, 2017; Shcherbatykh, Lykova, and Rogacheva, 2016; Poltavtsova and Poltavtsova, 2018).

Let us remind that the educational technology is a complex that includes educational models, diagnostic tools for current knowledge of learners, criteria for choosing the optimal model for these specific conditions, and ideas about the planned learning outcomes (Guzeev, 1996).

The analysis of many literary sources allowed us to point out the group of authors who most fully investigated fractal pedagogy's educational technologies (Abdullina, Majuga, and Sinicina, 2016). Table 1 reflects the list of educational technologies used in the framework of the fractal approach in teaching, supplemented by us, and describes the features of their application in practice - when passing through the elements of stochastics.

1. The research is experimental. In the course of the pedagogical experiment during the parallel study of theme "Statistical characteristics" by the seventh grades of the Moscow State Educational Institution "School No. 3 named after O. A. Morozov", the following technologies of the fractal approach were applied: learning in collaboration, vitagenic learning with the holographic method of projections, game, and web-quest technology.

2. After the lessons, both classes 7"A" and 7"B" went through a multimedia case with practical content during the extracurricular activities. When checking, two indicators were considered: the number of correct answers and the time spent on the task.

3. The proposed case was made in the seventh grades' cohort. The class 7"A" was already partially familiar with the educational technologies used in studying the "Statistical Characteristics," including the case technology.

4. The proposed case tasks were performed by 16 learners from the class 7"A" and 23 learners...
from the class 7"B." The total number of learners is 39.

5. The number of correct answers and the time spent on the task are two main criteria for evaluating.

Using the angular Fisher transformation, we verified the reliability of the obtained results, formulated hypotheses:

H₀: The main hypothesis says that the proportion of learners who got the correct answer to the tasks in the case is no more in the experimental group than in the control group.

H₁: The alternative hypothesis says that the proportion of learners who got the correct answer to the experimental group task is more significant than in the control group.

Let us analyze the obtained data, considering that there were no wrong answers in 7"A," and there were four wrong answers in 7"B." Let us move on to the results.

RESULTS

The process of implementing the pilot activities is shown in Table 2. The authors present the fractal approach's proven technologies in studying elements

<table>
<thead>
<tr>
<th>Lesson/topic of the lesson</th>
<th>Proven technologies of the fractal approach in education</th>
<th>The activity of the learners at the lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson No. 1. The arithmetic mean</td>
<td>Education technology in collaboration</td>
<td>After studying the new material, learners were divided into small groups of four people. Each group received an appropriate task from the textbook. The completed tasks were commented on by the small group leader and monitored by other learners.</td>
</tr>
<tr>
<td>7&quot;A&quot;</td>
<td>7&quot;B&quot;</td>
<td>Oct 03, 2019</td>
</tr>
</tbody>
</table>
| Lesson No. 2. Range | Webquest technology | The lesson is presented in three stages:  
- Initial stage. The learners learned the topic of "The Composition of the Russian Population in 105 Years", then they divided into teams, have the roles assigned to each other, and approved goals and objectives.  
- Main stage. Individual work was carried out to achieve a common team result. An ICT project was created, during which tasks were performed, the results were summed up.  
- Final stage. Conclusions and suggestions were formulated based on the results of the study.  
Both teacher and learners participated in the evaluation through discussion. |
| 7"A" | 7"B" | Oct 08, 2019 | Oct 08, 2019 |
| Lesson No. 3. Fashion | Game technology | One of the lesson elements was the "Tic-Tac-Toe" game, during which each schoolchild received a task to determine the most common subject or natural phenomenon shown in the presentation. |
| 7"A" | 7"B" | Oct 09, 2019 | Oct 10, 2019 |
| Lesson No. 4. Median as a statistical characteristic | Vitagenic education with a holographic projection method | The study of this topic was accompanied by an appeal to the life experience of learners. The learners provided examples of finding the middle of the segment, determining the middle of the school day, the largest number of people entering and leaving the bus’s middle door, etc. |
| 7"A" | 7"B" | Oct 10, 2019 | Oct 11, 2019 |

Implementing the case

Learners worked on the received case of "Energy Consumption in the World." They supplemented it with its material from other sources of information. The learners discussed the contents of the case, determined the values for comparison and construction of tables and graphs, and answered the problematic question.
of the probability-statistical line of the school course in mathematics.

The Fisher angular transformation was used to determine the significance of differences between the control and experimental samples' percentages with the correct answer to the case tasks. In the study course, the correct answer was considered to be "the effect presence," and the receipt of an incorrect answer was considered as "the lack of the effect."

The empirical value of the criterion is calculated by the formula:

\[ q_{emp} = (\varphi_1 - \varphi_2) \sqrt{\frac{n_1 \cdot n_2}{n_1 + n_2}}, \]  \hspace{1cm} (1)

where \( \varphi_1 \) and \( \varphi_2 \) are the angles corresponding to the percentages obtained in the control and experimental groups with a positive result (\( \varphi_1 > \varphi_2 \)), and \( n_1 \) and \( n_2 \) are the numbers of learners in each group.

The control group comprised of 23 learners of class 7"B." The experimental group comprised of 16 learners of class 7"A." The authors calculated the values of \( \varphi_1 \) and \( \varphi_2 \) for each group using the statistical table: \( \varphi_1(100\%) = 3.142 \) and \( \varphi_2(82.6\%) = 2.281 \).

Then: \( q_{emp} = (3.142 - 2.281) \sqrt{\frac{16 \cdot 23}{16 + 23}} = 0.861 \cdot 3.07 = 2.64 \).

Thus, the following results are obtained (Table 3):

The authors now turn to the temporary assessment of the results. In the experiment, the execution time of the case is a continuous random variable (X is the time of class 7"A," Y is the time of class 7"B"). Using the Pearson statistical criterion at a significance level of 0.05, the authors confirmed that random variables X and Y are distributed according to the normal law. The equality of the variances of these random variables was determined according to the Fisher criterion.

The authors now turn to the statistical Student's criterion. They will make the statement about the equality of the quantities X and Y's mathematical expectations as the main hypothesis. An alternative hypothesis is that the mathematical expectation of X is greater than the mathematical expectation of Y. Then: \( M(X) = M(Y), H_1: M(X) > M(Y) \).

The authors found \( t_{obs} \) using the following formula:

\[ t_{obs} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{(n-1) \cdot s_x^2 + (m-1) \cdot s_y^2}{n + m}}}, \]  \hspace{1cm} (2)

where \( n \) and \( m \) are the sizes of X and Y samples, \( \bar{x} \) and \( \bar{y} \) are the sample means, \( s_x^2 \) and \( s_y^2 \) are the corrected sample variances.

Table 4 presents the temporary results (in seconds) of the control and experimental groups' case execution.

Taking the information from this table, the authors calculate the necessary data:

<table>
<thead>
<tr>
<th>Groups</th>
<th>The effect is present</th>
<th>The effect is not present</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Qty of learners</td>
<td>% share</td>
</tr>
<tr>
<td>Experimental group, class 7&quot;A&quot;</td>
<td>16</td>
<td>100</td>
</tr>
<tr>
<td>Control group, class 7&quot;B&quot;</td>
<td>19</td>
<td>82.6</td>
</tr>
</tbody>
</table>

Figure 1: Significance axis (Compiled by the authors).
n=16; m=23; \( \bar{x} = 747.438; \bar{y} = 697.522; s_x^2 = 15,666.529; s_y^2 = 10,835.534 \).

The authors then calculate the experimental value of the Student’s criterion:

\[
\begin{align*}
t_{\text{obs}} &= \frac{747.438 - 697.522}{\sqrt{(16 - 1) \cdot 15666.529 + (23 - 1) \cdot 10835.534}} \\
&= \frac{747.438 - 697.522}{\sqrt{16 \cdot 23 \cdot (16 + 23 - 2)}} = 1.356.
\end{align*}
\]

The amount of degrees of freedom equals to \( f = n + m - 2 = 16 + 23 - 2 = 37 \). The right area is critical: \( t_{\text{kp}; +\infty} \). According to the Student’s table, the authors calculate the following: \( t_{\text{tab}}(n+m-2; \alpha) = t_{\text{tab}}(37; 0.05) \approx 1.684 \). \( t_{\text{kp}} \approx -1.684 \) (the right area is critical). The experimental value of the t-criterion fell into the critical area \( t_{\text{obs}} < t_{\text{kp}} \). Therefore, the null hypothesis should be rejected in favor of the alternative one.

The authors concluded that the discrepancy in the case's execution time at a significance level of 0.05 is significant. It follows that learners from the class 7"A" coped with the tasks of the case faster. Conclusion: the fractal approach technologies in the study of the "Statistical characteristics" topic affected stochastics' successful teaching. Below, the authors summarize the received data. Figure 2 shows the results obtained from the calculations of the Fisher angular transformation.

Timespan results of the case study by the experimental group are presented in Figure 3.

Timespan results of the case study by the control group are presented in Figure 4.

Figures 2-4 show that the learners from class 7"A" showed the best results.

<table>
<thead>
<tr>
<th>Timespan (in seconds)</th>
<th>Learners of 7&quot;A&quot;</th>
<th>Timespan (in seconds)</th>
<th>Learners of 7&quot;B&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>530-585</td>
<td>2</td>
<td>495-540</td>
<td>2</td>
</tr>
<tr>
<td>585-640</td>
<td>1</td>
<td>540-585</td>
<td>2</td>
</tr>
<tr>
<td>640-695</td>
<td>1</td>
<td>585-630</td>
<td>1</td>
</tr>
<tr>
<td>695-750</td>
<td>4</td>
<td>630-675</td>
<td>3</td>
</tr>
<tr>
<td>750-805</td>
<td>3</td>
<td>675-720</td>
<td>4</td>
</tr>
<tr>
<td>805-860</td>
<td>1</td>
<td>720-765</td>
<td>4</td>
</tr>
<tr>
<td>860-915</td>
<td>3</td>
<td>765-810</td>
<td>4</td>
</tr>
<tr>
<td>915-970</td>
<td>1</td>
<td>810-855</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 4: Distribution of Case Execution Time “Energy Consumption in the World” (Compiled by the Authors)

Figure 2: Comparative chart of the results of the case of the experimental and control groups (Experimental data transformed by the authors using volumetric histograms in Google services).
DISCUSSION

The use of the fractal approach in education is an urgent problem of our time. Many scientists and even only ascendant scientists are engaged in developing this problem's specific issues. This statement is confirmed by the works by Massel (2016), Madzhuga, Abdullina, Sinitsina, Askarova, and Filipenko (2017), Ostapenko (2019), Skopa (2019), Makarova (2016).

The educational process of general academic institutions - schools, which should be built on each of the lessons within the fractal approach framework - is not left without attention. The mathematics lessons are no exception, so in the article, we tried to point out the possible ways of implementing the fractal approach to one of the lines within the school mathematics course framework, which is the probabilistic-statistical line. Namely, the authors indicated a list of technologies that can be used in educational activities. In the study, the authors pointed out the popularity:

- Of educational technologies in studying. Educational technologies are studied in the works by the following authors: Ilyushenko (2018), Ivanova (2016), Golikova (2019).


- Health-saving technologies, which are studied by Bakshaeva and Kulikovskaya (2017), Kuznetsov (2017), Likhacheva and Koshevaya (2020).

- Game technologies, the didactic possibilities of which are described in the works by Abramova (2016), Kukhorenko (2016), Uminova (2016).
- Technologies of vitagenic learning with the holographic projection method, the research of which is presented by Morozova (2017), Mashukova and Shkarupa (2020), Teterich (2017).

- Reflexive evaluation technologies, the potential of which is given in the works of researchers, including Burdyugova (2017), Verevkin and Nadtochy (2016), Yunusbaev and Kazarbayev (2008).


- Case technologies, described in the works by Belousova (2016), Hajikurbanova (2016), Nezobroeva (2016).

The use of these technologies in the educational process is widely used in practice, as evidenced by numerous international publications.

In this regard, the conducted experimental research aimed at identifying the didactic potential of modern educational technologies in implementing the fractal approach in mathematics lessons has become necessary and confirmed by the obtained results.

As noted above, the seventh grade's parallel was passed a modern multimedia case "energy consumption in the world," where the number of correct answers and the time spent completing the task were the main evaluation criteria. The best result was in 7"A," which was already partially familiar with the teacher's learning technologies in math lessons when studying the topic "Statistical characteristics."

Thus, the fundamental factor in achieving the effectiveness of modern educational technologies used in mathematics lessons in the context of a fractal approach to learning was the improvement of knowledge on this topic, in particular, the selection and structuring of learning technologies, the development of a new teaching methodology and the procedure for evaluating students' knowledge.

In our view, the following provisions are the defining specifics of the modernization of key components of the pedagogical process based on the fractal approach with the use of modern educational technologies, which affects the effectiveness of the process of teaching mathematics:

- First, the proposed technologies of the fractal approach to teaching probabilistic-statistical lines contribute to more solid assimilation of complex mathematical knowledge, a better, more intensive assimilation of information perception, excluding the possibility of conceptual material's mechanical memorization; activate the establishment of interdisciplinary connections of mathematics with physics, chemistry, geography, biology, and other school subjects; help to conduct a simultaneous diagnostic procedure for intersubject connections within a single discipline.

- Second, the fractal approach in mathematics lessons and the modern educational technologies systematization allow control and management of the depth of established intersubject, inter-conceptual connections, and also allow, within their applicability limits, to satisfy learners' personal educational needs, allow the teacher to focus on the required educational material presentation depth and various profiling subjects.

- Third, the fractal approach technologies introduction to school educational practice makes it possible to increase significantly the accuracy and speed of assessing children's learning level, to identify hidden opportunities for students to comprehend knowledge, that is, the "hidden" potential objectively existing but having not yet been subjectively identified.

Later, the authors note that significant changes are currently taking place in the Russian education system. The main and primary task is the need to develop and implement the latest technologies and techniques in the learning process, which, in turn, will contribute to improving the efficiency of learning material by students and improving the quality of education throughout the country.
A general education school should form an integral system of universal knowledge, skills, and abilities systematized based on the use of a particular technology and the experience of independent activity of learners and students' responsibility, which determines the modern quality of educational content. Improving the quality of education should be carried out not at the expense of an additional load on students, but through the improvement of forms and methods of teaching, the selection of educational content, through the introduction of educational technologies focused not so much on the transfer of ready-made knowledge, but on the formation of a complex of personal qualities of students. Such educational technologies, from our point of view, are technologies of the fractal approach to learning.

The use of educational technologies in the context of a fractal approach makes it possible to solve educational tasks and form children's readiness for independent knowledge of the world around them.

Focusing efforts on improving the quality and effectiveness of educational and educational work, it is necessary to ensure that each lesson contributes to the development of students' cognitive interests, activity, and creativity, and, consequently, to improve the quality of teaching. Therefore, it is advisable to use in the lesson: vitagenic teaching technology with a holographic projection method, fractal and neural network technologies, cooperation technology, humanitarian technologies, portfolio technology, case technology, web quest technology, reflective-evaluative, game, and health-saving technologies.

The fractal approach's listed modern educational technologies can be considered a new way of transferring knowledge that corresponds to qualitatively new learning and student development content. This method allows the student to study with interest, find sources of information, foster independence and responsibility in acquiring new knowledge, and develop the intellectual activity. Information and communication technologies can replace almost all traditional technical means of education. In many cases, this replacement is more effective. It allows the teacher to combine various tools, methods, and forms of learning that contribute to deeper and more conscious assimilation of the material being studied, save lesson time, and make the lesson more informative. In this regard, it is quite natural to introduce technologies of the fractal approach into the modern educational process. However, our research can be expanded and continued through the development and discovery of other educational technologies that can be applied within the fractal approach framework in lessons not only in mathematics but also in other school subjects. Further discoveries will allow us to present an entire system, the functional components of which are technologies of the fractal approach to learning.

CONCLUSION

The problem of applying the fractal approach technologies in stochastic learning was solved in the study. "Statistical Characteristics" is the course taught to learners as part of a new educational paradigm. While learning the mentioned course, the authors involved educational technology in collaboration, vitagenic learning technology with a holographic projection method, web quest, and game technology. Evaluation of results involved case technology.

Thus, in the study, educational technologies of the fractal approach in education were presented. The possibilities of their application in studying the probabilistic-statistical line were described, and their effectiveness was shown by the method of statistical processing of quantitative results. Thus, the requirements of the primary goal of scientific research were met.

LIMITATIONS AND STUDY FORWARD

The limitations of scientific research are experimenting in only one parallel of a comprehensive school, considering the fractal approach's technologies as applied to one branch of mathematics.

It is supposed to continue to work in order to get rid of the limitations of this study. First, it will be necessary to conduct a more extensive study, taking into account not one parallel of the general educational institution, but several. The authors see the prospect of further studying the technologies of the fractal approach in teaching mathematics to learners in a more detailed examination of each technology separately and its impact on the educational process, which we will build under the new education paradigm. In addition to the above, one can expand the list of educational technologies and develop criteria based on which to present the technology of the fractal approach in training in the form of a specific fractal system.

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