Supporting Materials for "Adjusting complex heterogeneity in treatment assignment in observational studies" by authors Yu, Zhou, Liu , Zou and Willke.

Simulation Study

Treatment assignment generated from model (T_1) We choose the parameters $\theta_1 = -0.04(Age)$, $\theta_2 = 0.5(Sev)$, $\theta_3 = 0.7(Test)$, $\theta_4 = 0.8(Insu)$ to get $\Pr(Trt = 1) = 0.5$. The simulation results are summarized in Table 1.

Table 1 is about here

Treatment assignment generated from model (T_2) We choose the parameters $\theta_1 = -0.04(Age)$, $\theta_2 = 0.5(Sev)$, $\theta_3 = 0.7(Test)$, $\theta_4 = 0.8(Insu)$ to get $\Pr(Trt = 1) = 0.5$. The simulation results are summarized in Table 2.

Table 2 is about here

Treatment assignment generated from model (T_3) In order to get $\Pr(Trt = 1) = 0.5$, we consider the following treatment assignment:

$$Trt_{ij} = \begin{cases} Bern(1, p = 0.8 + \mu_i), Insu_{ij} = 1 \,\&\, \eta_i \geq 0.4 \\ 0, (Insu_{ij} = 0 or \eta_i < 0.4) \,\&\, Age_{ij} \in (20 + \xi_i, 40) \,\&\, Sev_{ij} < 1 \\ 1, (Insu_{ij} = 0 or \eta_i < 0.4) \,\&\, Age_{ij} \in (20 + \xi_i, 40) \,\&\, Sev_{ij} \geq 1 \\ 0, (Insu_{ij} = 0 or \eta_i < 0.4) \,\&\, Age_{ij} \leq 20 + \xi_i \,\&\, Sev_{ij} < 1 \\ Bern(1, p = 0.7 + \mu_i), (Insu_{ij} = 0 or \eta_i < 0.4) \,\&\, Age_{ij} \leq 20 + \xi_i \,\&\, Sev_{ij} \geq 1 \\ 0, (Insu_{ij} = 0 or \eta_i < 0.4) \,\&\, Age_{ij} \geq 40 \,\&\, Sev_{ij} < 1 \\ Bern(1, p = 0.8 + \mu_i), (Insu_{ij} = 0 or \eta_i < 0.4) \,\&\, Age_{ij} \geq 40 \,\&\, Sev_{ij} \geq 1 \end{cases}$$

The simulation results are summarized in Table 3.

Table 3 is about here

Table 1: Treatment assignment model (T_1)

	$b=0$				$b=1$			
	$\alpha=0$		$\alpha=0.5$		$\alpha=0$		$\alpha=0.5$	
Method	Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\hat{\alpha}_{S\cdot I}$	0.142	0.055	0.642	0.055	0.141	0.109	0.641	0.109
$\hat{\alpha}_{S\cdot M}$	0.140	0.066	0.640	0.066	0.139	0.116	0.639	0.116
$\hat{\alpha}_{S\cdot D}$	0.140	0.082	0.639	0.082	0.141	0.844	0.641	0.844
$\hat{\alpha}_{IPW\cdot I}$	0.008	0.204	0.506	0.208	0.007	0.224	0.506	0.228
$\hat{\alpha}_{IPW\cdot M}$	-0.196	0.531	0.346	0.542	-0.197	0.537	0.346	0.548
$\hat{\alpha}_{IPW\cdot D}$	-0.017	0.470	0.484	0.479	-0.020	0.492	0.487	0.478
$\hat{\alpha}_{I\cdot I}$	0.002	0.035	0.502	0.035	0.844	0.121	1.344	0.121
$\hat{\alpha}_{I\cdot M}$	0.002	0.035	0.502	0.035	0.199	0.065	0.699	0.065
$\hat{\alpha}_{I\cdot D}$	0.006	0.059	0.506	0.059	0.006	0.059	0.506	0.059
$\hat{\alpha}_{M\cdot I}$	0.017	0.054	0.517	0.054	0.852	0.132	1.352	0.132
$\hat{\alpha}_{M\cdot M}$	0.017	0.053	0.517	0.053	0.210	0.046	0.710	0.046
$\hat{\alpha}_{M\cdot D}$	0.017	0.041	0.517	0.041	0.017	0.041	0.570	0.041
$\hat{\alpha}_{D\cdot I}$	0.000	0.037	0.500	0.037	0.000	0.038	0.500	0.038
$\hat{\alpha}_{D\cdot M}$	0.000	0.037	0.500	0.037	0.000	0.038	0.500	0.038
$\hat{\alpha}_{D\cdot D}$	0.000	0.037	0.500	0.037	0.000	0.037	0.500	0.037
$\hat{\alpha}_{PLS\cdot I}$	0.001	0.035	0.501	0.035	0.824	0.115	1.324	0.115
$\hat{\alpha}_{PLS\cdot M}$	0.001	0.035	0.501	0.035	0.021	0.036	0.521	0.036
$\hat{\alpha}_{PLS\cdot D}$	0.001	0.037	0.501	0.037	0.001	0.037	0.501	0.037

Note: $\left(\hat{\alpha}_{S\cdot I},\hat{\alpha}_{S\cdot D},\hat{\alpha}_{S\cdot M}\right)$: The PS stratification estimates. $\left(\hat{\alpha}_{IPW\cdot I},\hat{\alpha}_{IPW\cdot D},\hat{\alpha}_{IPW\cdot M}\right)$: PS inverse probability weight estimates. $\hat{\alpha}_{\cdot\cdot}$ PS regression with PS from the model of using ignoring (I), dummy variable (D) and mixed model (M) to adjust the heterogeneity in propensity score and the second subscript of $\hat{\alpha}$ denotes the response from the model of using ignoring (I), dummy variable (D) and mixed model (M) to adjust the heterogeneity in response. $\hat{\alpha}_{PLS\cdot\cdot}$: the PLS method and the second denotes the response model.

Table 2: Treatment assignment model (T_2)

	$b=0$				$b=1$			
	$\alpha=0$		$\alpha=0.5$		$\alpha=0$		$\alpha=0.5$	
Method	Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\hat{\alpha}_{S\cdot I}$	0.142	0.055	0.642	0.055	0.964	0.123	1.464	0.123
$\hat{\alpha}_{S\cdot M}$	0.140	0.066	0.640	0.066	0.966	0.128	1.466	0.128
$\hat{\alpha}_{S\cdot D}$	0.140	0.082	0.640	0.082	0.234	0.084	0.734	0.084
$\hat{\alpha}_{IPW\cdot I}$	0.008	0.204	0.506	0.208	0.857	0.237	1.355	0.241
$\hat{\alpha}_{IPW\cdot M}$	-0.196	0.531	0.346	0.542	0.803	0.541	1.345	0.554
$\hat{\alpha}_{IPW\cdot D}$	-0.017	0.470	0.484	0.479	-0.020	0.492	0.482	0.500
$\hat{\alpha}_{I\cdot I}$	0.002	0.035	0.502	0.035	0.001	0.102	0.501	0.102
$\hat{\alpha}_{I\cdot M}$	0.002	0.035	0.502	0.035	0.005	0.055	0.505	0.055
$\hat{\alpha}_{I\cdot D}$	0.006	0.059	0.506	0.059	0.006	0.059	0.506	0.059
$\hat{\alpha}_{M\cdot I}$	0.014	0.053	0.514	0.053	0.012	0.110	0.512	0.110
$\hat{\alpha}_{M\cdot M}$	0.014	0.051	0.514	0.051	0.016	0.046	0.516	0.046
$\hat{\alpha}_{M\cdot D}$	0.016	0.043	0.516	0.043	0.016	0.043	0.516	0.043
$\hat{\alpha}_{D\cdot I}$	0.000	0.037	0.500	0.037	0.000	0.037	0.500	0.037
$\hat{\alpha}_{D\cdot M}$	0.000	0.037	0.500	0.037	0.000	0.037	0.500	0.037
$\hat{\alpha}_{D\cdot D}$	0.000	0.037	0.500	0.037	0.000	0.037	0.500	0.037
$\hat{\alpha}_{PLS\cdot I}$	0.001	0.035	0.501	0.035	0.005	0.102	0.505	0.102
$\hat{\alpha}_{PLS\cdot M}$	0.001	0.035	0.501	0.035	0.001	0.036	0.501	0.036
$\hat{\alpha}_{PLS\cdot D}$	0.001	0.037	0.501	0.037	0.001	0.037	0.501	0.037

Note: the notations are the same as in Table 1.

Table 3: Treatment assignment model (T_3)

	$b=0$				$b=1$			
	$\alpha=0$		$\alpha=0.5$		$\alpha=0$		$\alpha=0.5$	
Method	Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\hat{\alpha}_{S\cdot I}$	0.039	0.616	0.539	0.616	0.619	0.643	1.119	0.634
$\hat{\alpha}_{S\cdot M}$	0.017	0.610	0.517	0.610	0.605	0.633	1.105	0.633
$\hat{\alpha}_{S\cdot D}$	0.337	0.536	0.837	0.536	0.686	0.602	1.186	0.602
$\hat{\alpha}_{IPW\cdot I}$	-16.836	3.138	-16.422	3.140	-16.495	3.149	-16.082	3.152
$\hat{\alpha}_{IPW\cdot M}$	-27.018	6.627	-26.590	6.626	-26.649	6.584	-26.220	6.584
$\hat{\alpha}_{IPW\cdot D}$	-38.345	12.583	-37.987	12.589	-40.675	13.591	-40.317	13.596
$\hat{\alpha}_{I\cdot I}$	-0.068	0.044	0.432	0.044	0.458	0.089	0.958	0.089
$\hat{\alpha}_{I\cdot M}$	-0.068	0.045	0.432	0.045	0.038	0.064	0.538	0.064
$\hat{\alpha}_{I\cdot D}$	-0.077	0.065	0.423	0.065	-0.077	0.065	0.423	0.065
$\hat{\alpha}_{M\cdot I}$	0.011	0.063	0.511	0.063	0.542	0.109	1.042	0.109
$\hat{\alpha}_{M\cdot M}$	0.011	0.062	0.511	0.062	0.123	0.062	0.623	0.062
$\hat{\alpha}_{M\cdot D}$	0.008	0.056	0.508	0.056	0.008	0.056	0.508	0.056
$\hat{\alpha}_{D\cdot I}$	-0.089	0.049	0.411	0.049	-0.118	0.050	0.382	0.050
$\hat{\alpha}_{D\cdot M}$	-0.089	0.049	0.411	0.049	-0.101	0.049	0.399	0.049
$\hat{\alpha}_{D\cdot D}$	-0.095	0.049	0.405	0.049	-0.095	0.049	0.405	0.049
$\hat{\alpha}_{PLS\cdot I}$	0.000	0.039	0.500	0.039	0.450	0.070	0.950	0.070
$\hat{\alpha}_{PLS\cdot M}$	0.000	0.039	0.500	0.039	0.011	0.040	0.511	0.040
$\hat{\alpha}_{PLS\cdot D}$	0.001	0.040	0.501	0.040	0.001	0.041	0.501	0.040

Note: the notations are the same as in Table 1.