Process Capability Indices for Processes when the Underlying Data are Interval-Valued

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Abstract: One of the important activities of process quality management is to see that the processes of interest are, in fact, stable and capable. In this paper, the problem of obtaining process capability indices (PCIs) for the processes when the underlying data are interval-valued is considered. Since interval-valued data such as systolic and diastolic readings have specifications for both lower and upper values, drawing PCIs cannot be straightforward. In this paper, we attempted to build connections between the lower and upper specifications limits based on which the resulting PCIs are drawn. This is done by considering the coefficients of inflation and the mean shift values of distributions of both lower and upper values of the interval-valued data. The new expressions for the proposed PCIs are determined. We have considered the systolic and diastolic data to demonstrate the computations of PCIs.

Keywords: Interval-valued data, process capability indices, coefficients of inflation, systolic and diastolic data.

1. INTRODUCTION

In their recent research Xiong and Feng (2022) [1] discussed the role of process quality management that is to see that the processes of interest are not only stable but capable as well. In the study of statistical process control (SPC), once it is established that a process is in statistical control, an important follow-up activity is to check the capability of the process. For example, in healthcare setup, such capability studies are often applied to data collected from laboratory tests, mainly to ensure that the results are well within the established standards or specifications. Similarly, in a manufacturing setup, a quality practitioner may be interested to know and quantify how much the process is capable of producing the product given the product specifications that reflect the quality characteristics of the underlying process. It may be recalled that the term statistical quality control (SQC) refers to using statistical techniques for measuring and improving the quality of processes and includes SPC in addition to other techniques, such as sampling techniques, experimental designs, variation reduction techniques, control charts, process capability analysis, and process improvement methods. In SQC studies, quality control charts containing control limits are used to monitor if a product or a process is under statistical control.

The capability of a process is often quantified by process capability indices (PCIs) in order to know if there exist a link between control and specification limits [2]. The *Quality Glossary* compiled by Bemowski (1992) [3], includes the definition of process capability which states that process capability is considered as a statistical measure of the inherent variability of the process for a given quality characteristic. A careful look at this definition reveals the fact that process capability study is, in fact, effective if the process is in statistical control. PCIs are commonly used in the industries such as manufacturing, healthcare and other service industries for obtaining quantitative measures on process potential and performance. For example, Faud *et al*. (2020) [4] considered a study of PCIs to identify the problems associated with the dialysis unit with an aim to provide recommendations for further investigation and analysis. Wooluru *et al*. (2016) [5] considered data on bore diameters to study the resistivity of silicon wafers during the boring process.

Kotz and Johnson (2002) [6] provided a compact literature survey and brief interpretations on as many publications on PCIs published during 1992-2002. This literature survey also presents an assessment of the most widely used PCIs from the perspectives of normal, nonnormal, univariate and multivariate cases. As pointed out by Kotz and Johnson (2002) [6], the PCIs are, in fact, intended to provide single-number assessments of ability to meet specification limits on quality characteristic(s) of interest. Bothe (1999) [7] studied the aspects of developing capability indices for multiple process streams. Vannman and Albing (2007) [8] explored the process capability plots for process with one-sided specification limits. Xiong and Feng (2022) [1] proposed a set of new metrics called process quality indices which can measure quality capability of the process with zero-loss baseline adapted from Taguchi's quality loss function model. Alatefi *et al*. (2023) [9] proposed a new approach for process

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capability analysis in which multivariate quality characteristics are used.

Notwithstanding, from literature it is observed that there exist conflicting opinions with regard to the use of control limits and specification limits [2]. Accordingly, while control limits and specification limits are often considered as independent entities, they are, in fact, imposed upon the same process/product. One can see that there are as many studies that worked on the development of either control limits [10-15] or process capability indices [8, 16-22]. One may be interested to note that while specification limits are used to determine PCIs, they are, of course, not considered for the construction of control limits until Carr (1989) [23] proposed modified control charts by linking control limits and specification limits. Ravichandran (2019) [2] conducted a review of specification limits and control limits by linking the same to Six Sigma quality.

It may be noted that while statistical analyses are always straightforward if the data are real-valued, complexities do exist if the observed data are interval-valued. This is due to the reason that there are situations where only interval-valued data are available in the form of ranges rather than single values. Often, this is necessitated in advanced data collection processes and machine learning techniques. While statistical analyses of products/processes based on interval-valued data are scarce there are some studies that applied linear and non-linear regression methods for the interval-valued data prediction tasks. Al-Asadi (2022) [24] presented a review of some important methods that are used to analyse interval-valued data. Hsu *et al*. (2013) [25] attempted to develop control charts for process with interval-valued data, Ravichandran *et al*. (2024) [26] developed control charts by combining Taguchi and Six Sigma approaches when the data are observed as real valued.

In order to check if any related studies on PCIs for interval-valued data are done in the literature, we used the phrase ‘process capability indices for interval valued data’ as keyword to search using the google search engine. However, the search has not returned any results on the availability of publications in this field of research and we assume that we are the first one to start such a study. In this article, we use the multistream probability distributions approach to consider the information from both lower and upper values of the interval-valued data to develop procedures for obtaining the required PCIs. Accordingly, the concepts of inflation coefficient and mean shifts are used to determine the PCIs representing specifications of both lower and upper values of the interval-valued data. The results are numerically evaluated and we tried to demonstrate these indices with appropriate real time example that considers interval-valued data on systolic pressure and diastolic pressure to study their influence on the pulse rate.

The remainder of the paper is organized as follows: In section 2 we reviewed and presented some commonly used PCIs. The approach to determine the proposed PCIs is described in Section 3. After discussing the one-sided and two-sided PCIs, a summary of the expressions obtained for PCIs in terms of shift and inflation coefficients is give in Section 3. A real-time example comprising systolic and diastolic data is considered in Section 4 to demonstrate the construction PCIs using the proposed approach. Final discussions and conclusions are given in Section 5.

2. SOME COMMONLY USED PROCESS CAPABILITY INDICES

As discussed earlier, researchers such as Sullivan (1985) [16], Kane (1986) [17], Gunter (1989) [27], Pearn *et al*. (1992) [18] Kotz and Johnson (1993) [19], Jeang (1997) [20], Kotz and Lovelace (1998) [21], Pearn and Chen (2002) [22], Vannman and Albing (2007) [8], Ravichandran (2019) [2], Alatefi *et al*. (2023) [9] have contributed much to the studies on PCIs. The development of PCIs has covered the process situations under univariate and multivariate distributions. Other than normal and nonnormal process situations, the cases of such processes with one sided and two sided specification settings are also considered by various authors for studying PCIs.

Here, we present some of the commonly used PCIs: (i) based on two sided specification limits notationally given as C_p, C_{pk}, C_{pm} and C_{pmk} (Sullivan, 1985) [16]. and (ii) based on one sided specification limits given as C_{pkl}, C_{pku}, C_{pml} and C_{pmu} (Pearn and Chen, 2002; Vannman and Albing, 2007) [8, 22], where

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

$$C_{pk} = min\left\{\frac{\bar{x} - LSL}{3\sigma}, \quad \frac{USL - \bar{x}}{3\sigma}\right\} \tag{2}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\bar{x} - T)^2 + \sigma^2}} \tag{3}$$

$$C_{pmk} = min\left\{ \frac{T - LSL}{3\sqrt{(\bar{x} - T)^2 + \sigma^2}}, \quad \frac{USL - T}{3\sqrt{(\bar{x} - T)^2 + \sigma^2}} \right\}$$

$$C_{pkl} = \frac{\bar{x} - LSL}{3\sigma}, \quad C_{pku} = \frac{USL - \bar{x}}{3\sigma} \tag{4}$$

$$C_{pml} = \frac{T - LSL}{3\sqrt{(\bar{x} - T)^2 + \sigma^2}}, \quad C_{pmu} = \frac{USL - T}{3\sqrt{(\bar{x} - T)^2 + \sigma^2}} \tag{5}$$

Where

LSL: Lower Specification Limit of the quality characteristic X

USL: Upper Specification limit of the quality characteristic X

T: Target value of the quality characteristic X

σ^2: Population variance of the quality characteristic X

$\bar{x}$: Process mean of the quality characteristic X base on a sample of size n

It may be noted that if the population variance σ^2 is unknown, then the process variance $\hat{\sigma}^2$ of the quality characteristic X base on a sample of size n can be used.

For nonnormal case, the PCIs C_p and C_{pk} have been proposed by Clements (1989) [28] using percentiles and the expressions are given as follows:

$$C_p = \frac{USL - LSL}{P_{0.99865} - P_{0.00135}} \tag{6}$$

$$C_{pk} = min\left\{ \frac{USL - P_{0.5}}{P_{0.99865} - P_{0.5}}, \quad \frac{P_{0.5} - LSL}{P_{0.5} - P_{0.00135}} \right\} \tag{7}$$

Where P_α is the value corresponding to 100α percentile.

Recently, Xiong and Feng (2022) [1] developed a set of new metrics called process quality indices (PQIs) which can measure quality capability of the process with zero-loss baseline adapted from Taguchi's quality loss function model. The PQI equivalents for the PCIs C_p and C_{pk} are notationally given as Q_p and Q_{pk}, and the expressions are developed for the nominal-the-best, smaller-the-better and larger-the-better cases. Readers are referred to Xiong and Feng (2022) for detail on the computations of these PQIs.

It may be noted that while all these studies on PCIs and PQIs are based on the real-valued data, the use of these formulas are not straight forward when the data are interval-valued. In this work we have proposed to study the indices given in Eqns. (1) through (5) when the available data are interval-valued. Particularly, when the lower and upper values of the interval valued data are having separate LSL and USL values. For example, while systolic and diastolic values observed at a time is interval-valued, they have individual specifications as well. That is systolic has its own lower and upper specification limits and diastolic has its own lower and upper specification limits. In our work, we propose to determine various process capability indices using the multistream probability distributions. Such an approach is able to combine the means and variances of both the lower and upper values with respect to their specification limits, and hence the resulting indices values are reasonable and effective as well.

3. PROPOSED APPROACH TO DETERMINE PROCESS CAPABILITY INDICES

Carr (1989) and Hill (1956) [23, 29] opined that the process specifications, in fact, have a strong engineering basis for the quality of a product in the sense that only a negligible number of measurements of the quality characteristic of the product units shall fall outside the specifications limits. If the random variable X of the measurements of the quality characteristic is assumed to follow normal distribution, i.e., $X \sim N(\mu, \sigma)$, X may have its own LSL, T and USL, and hence we can have the estimates of μ and σ as follows:

$$\mu = T = \frac{LSL + USL}{2} \tag{8}$$

$$\sigma = \hat{\sigma} = \frac{USL - LSL}{k} \tag{9}$$

For example, for a Six Sigma process, we have $k = 12$ which has the probability 2×10^{-9} for the units (i.e., only a negligible number of units) to fall outside the specification limits. This implies that, if x_i is the i^{th} measurement of X, then we have

$$P(LSL \le x_i \le USL) = 1 - 2x10^{-9} \tag{10}$$

Now, let us consider an interval-valued random variable $X = [X_l, X_u]$ where X_l is the lower value and X_u is the upper value of the of a quality characteristic X. Further, let us assume that while X_l has the specification limits LSL_l, T_l and USL_l, X_u has the specification limits LSL_u, T_u and USL_u. With normality assumption, if we let $X_l \sim N(\mu_l, \sigma_l^2)$ and $X_u \sim N(\mu_u, \sigma_u^2)$, then using the Equations (9) and (10), the means and standard deviations are estimated as follows:

$$\mu_l = T_l = \frac{LSL_l + USL_l}{2}, \sigma_l = \hat{\sigma}_l = \frac{USL_l - LSL_l}{k_l} \quad (11)$$

$$\mu_u = T_u = \frac{LSL_u + USL_u}{2}, \sigma_u = \hat{\sigma}_u = \frac{USL_u - LSL_u}{k_u} \quad (12)$$

If $X_a = (X_l + X_u)/2$, then according to the property of distribution of sum of two normal random variables, we have $X_a \sim N(\mu_a, \sigma_a^2)$, where $\mu_a = (\mu_l + \mu_u)/2$ and $\sigma_a^2 = (\sigma_l^2 + \sigma_u^2 + 2\sigma_{lu})/4$ with $\sigma_{lu} = Cov(X_l, X_u)$. Refer to Figure **1** for graphical representation of the distributions of lower and upper values of interval-valued data and their combined distribution. If there is a sample of size n with interval-valued observations $[x_{li}, x_{ui}]$, $i = 1,2,\cdots\cdots,n$, then the mean μ_a and variance σ_a^2 can also be estimated as follows [30, 31]:

$$\hat{\mu}_a = \frac{1}{2m}\sum_{i=1}^{m}(x_{li} + x_{ui}) \quad (13)$$

$$\hat{\sigma}_a^2 = \frac{1}{4m}\sum_{i=1}^{m}(x_{li} + x_{ui})^2 - \frac{1}{4m^2}\left(\sum_{i=1}^{m}(x_{li} + x_{ui})\right)^2 \quad (14)$$

However, in case of control charts, most of the time several subsamples are considered. Let $X_{ij} = [x_{ij}^l, x_{ij}^u]$ be the interval-valued data corresponds to j^{th} interval-valued observation of i^{th} subsample, where x_{ij}^l is the lower value and x_{ij}^u is the upper value of the interval. With $i = 1,2,\cdots\cdots,m$ and $i = 1,2,\cdots\cdots,n$, the estimates of mean and variance can also be computed as it is given in Eqns. (15) and (16). That is,

$$\hat{\mu}_a = \frac{1}{2m}\sum_{i=1}^{m}\left(\bar{x}_i^l + \bar{x}_i^u\right) \quad (15)$$

$$\hat{\sigma}_a^2 = \frac{1}{4m}\sum_{i=1}^{m}(\bar{x}_i^l + \bar{x}_i^u)^2 - \frac{1}{4m^2}\left(\sum_{i=1}^{m}(\bar{x}_i^l + \bar{x}_i^u)\right)^2 \quad (16)$$

Where

$$\bar{x}_i^l = \frac{1}{n}\sum_{j=1}^{n}x_{ij}^l, \ \bar{x}_i^u = \frac{1}{n}\sum_{j=1}^{n}x_{ij}^u, \ i = 1,2,\cdots\cdots,m \quad (17)$$

Note that, we can also have $\hat{\mu}_a = \frac{\hat{\mu}_l + \hat{\mu}_u}{2}$, where $\hat{\mu}_l = \bar{x}_l = \frac{1}{m}\sum_{i=1}^{m}\bar{x}_i^l$ is an unbiased estimate of μ_l and $\hat{\mu}_u = \bar{x}_u = \frac{1}{m}\sum_{i=1}^{m}\bar{x}_i^u$ is an unbiased estimate of μ_u.

With $X_a \sim N(\mu_a, \sigma_a^2)$, we assume that $LSL_l = LSL$, $T = \mu_a$ and $USL_u = USL$, and hence

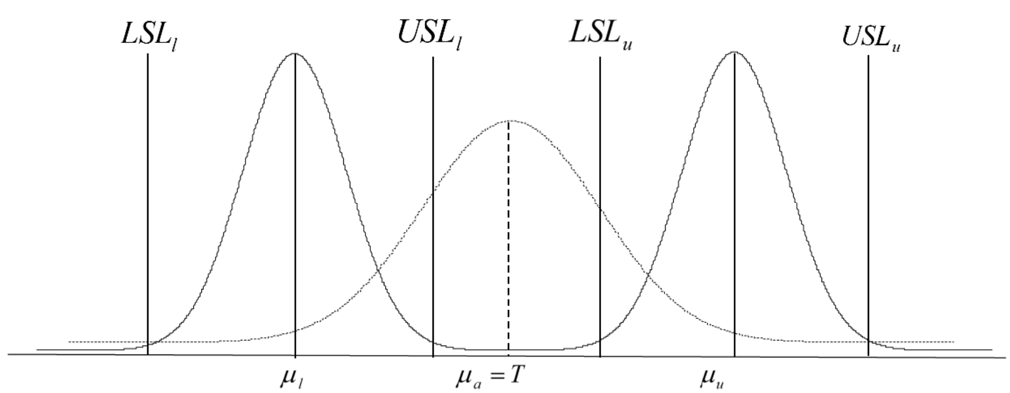

Figure 1: Normal distributions of lower and upper values of interval-valued data with specification limits (middle one is the combined distribution).

LSL_u and USL_l become redundant. Now, for some $k_{l1}, k_{l2}, k_{u1}, k_{u2} > 0$, we can write

$$\mu_a = \mu_l + k_{l1}\sigma_l = LSL_l + k_{l2}\sigma_l \text{ or}$$
$$\mu_a = \mu_u - k_{u1}\sigma_u = USL_u - k_{u2}\sigma_u \quad (18)$$

$$\Rightarrow \mu_l + k_{l1}\sigma_l = \mu_u - k_{u1}\sigma_u$$
$$\Rightarrow \mu_u - \mu_l = k_{l1}\sigma_l + k_{u1}\sigma_u \quad (19)$$

$$USL_u - LSL_l = k_{l2}\sigma_l + k_{u2}\sigma_u \quad (20)$$

Therefore, using Eqn. (1), the process capability index C_p can be given as

$$C_p = \frac{USL_u - LSL_l}{6\sigma_a} = \frac{k_{l2}\sigma_l + k_{u2}\sigma_u}{6\sigma_a} = \frac{1}{6}\left(\frac{k_{l2}}{c_{al}} + \frac{k_{u2}}{c_{au}}\right) \quad (21)$$

where, $c_{al} = \dfrac{\sigma_a}{\sigma_l}$ and $c_{au} = \dfrac{\sigma_a}{\sigma_u}$ (22)

are known as the inflation coefficients with regard to the distributions of lower value and upper values respectively. Such inflation coefficients, in fact, reflect the level of shift in the process [32, 33].

3.1. Two-Sided Process Capability Indices

Using Eqn. (2), the two-sided process capability index C_{pk} can be obtained as

$$C_{pk} = min\left\{\frac{\hat{\mu}_a - LSL_l}{3\sigma_a}, \quad \frac{USL_u - \hat{\mu}_a}{3\sigma_a}\right\}$$

$$= min\left\{\frac{\hat{\mu}_a - \mu_a + k_{l2}\sigma_l}{3\sigma_a}, \quad \frac{\mu_a + k_{u2}\sigma_u - \hat{\mu}_a}{3\sigma_a}\right\}$$

$$= min\left\{\frac{\hat{\mu}_a - \mu_a}{3\sigma_a} + \frac{k_{l2}}{3c_{al}}, \quad \frac{-(\hat{\mu}_a - \mu_a)}{3\sigma_a} + \frac{k_{u2}}{3c_{au}}\right\} \quad (23)$$

$$= min\left\{\frac{(\hat{\mu}_l - \mu_l) + (\hat{\mu}_u - \mu_u)}{6\sigma_l c_{al}} + \frac{k_{l2}}{3c_{al}}, \quad \frac{-[(\hat{\mu}_l - \mu_l) + (\hat{\mu}_u - \mu_u)]}{6\sigma_u c_{au}} + \frac{k_{u2}}{3c_{au}}\right\}$$

Following this approach, let us now obtain the another two-sided capability indices C_{pm} and C_{pmk} for interval-valued data as follows:

$$C_{pm} = \frac{USL_u - LSL_l}{6\sqrt{(\hat{\mu}_a - \mu_a)^2 + \sigma_a^2}}$$

$$= \frac{k_{l2}\sigma_l + k_{u2}\sigma_u}{3\sqrt{(\hat{\mu}_l - \mu_l)^2 + (\hat{\mu}_u - \mu_u)^2 + 2(\hat{\mu}_l - \mu_l)(\hat{\mu}_u - \mu_u) + 4\sigma_a^2}}$$

$$= \frac{k_{l2}/c_{al} + k_{u2}/c_{au}}{3\sqrt{\left(\frac{\hat{\mu}_l - \mu_l}{\sigma_l c_{al}}\right)^2 + \left(\frac{\hat{\mu}_u - \mu_u}{\sigma_u c_{au}}\right)^2 + \frac{2(\hat{\mu}_l - \mu_l)(\hat{\mu}_u - \mu_u)}{\sigma_a^2} + 4}} \quad (24)$$

$$C_{pmk} = min\left\{\frac{\mu_a - LSL_l}{3\sqrt{(\hat{\mu}_a - \mu_a)^2 + \sigma_a^2}}, \quad \frac{USL_u - \mu_a}{3\sqrt{(\hat{\mu}_a - \mu_a)^2 + \sigma_a^2}}\right\}$$

$$= min\left\{\frac{k_{l2}\sigma_l}{3\sqrt{(\hat{\mu}_a - \mu_a)^2 + \sigma_a^2}}, \quad \frac{k_{u2}\sigma_u}{3\sqrt{(\hat{\mu}_a - \mu_a)^2 + \sigma_a^2}}\right\}$$

$$= min\left\{\frac{2k_{l2}/c_{al}}{3\sqrt{\left(\frac{\hat{\mu}_l - \mu_l}{\sigma_l c_{al}}\right)^2 + \left(\frac{\hat{\mu}_u - \mu_u}{\sigma_u c_{au}}\right)^2 + \frac{2(\hat{\mu}_l - \mu_l)(\hat{\mu}_u - \mu_u)}{\sigma_l\sigma_u c_{al}c_{au}} + 4}}, \quad \frac{2k_{u2}/c_{au}}{3\sqrt{\left(\frac{\hat{\mu}_l - \mu_l}{\sigma_l c_{al}}\right)^2 + \left(\frac{\hat{\mu}_u - \mu_u}{\sigma_u c_{au}}\right)^2 + \frac{2(\hat{\mu}_l - \mu_l)(\hat{\mu}_u - \mu_u)}{\sigma_l\sigma_u c_{al}c_{au}} + 4}}\right\} \quad (25)$$

3.2. One-Sided Process Capability Indices

Now, the one-sided process capability indices C_{pkl}, C_{pku}, C_{pml} and C_{pmu} are given as

$$C_{pkl} = \frac{(\hat{\mu}_l - \mu_l) + (\hat{\mu}_u - \mu_u)}{6\sigma_l c_{al}} + \frac{k_{l2}}{3c_{al}} \quad (26)$$

$$C_{pku} = \frac{-[(\hat{\mu}_l - \mu_l) + (\hat{\mu}_u - \mu_u)]}{6\sigma_u c_{au}} + \frac{k_{u2}}{3c_{au}} \quad (27)$$

$$C_{pml} = \frac{2k_{l2}/c_{al}}{3\sqrt{\left(\frac{\hat{\mu}_l - \mu_l}{\sigma_l c_{al}}\right)^2 + \left(\frac{\hat{\mu}_u - \mu_u}{\sigma_u c_{au}}\right)^2 + \frac{2(\hat{\mu}_l - \mu_l)(\hat{\mu}_u - \mu_u)}{\sigma_l\sigma_u c_{al}c_{au}} + 4}} \quad (28)$$

$$C_{pmu} = \frac{2k_{u2}/c_{au}}{3\sqrt{\left(\frac{\hat{\mu}_l - \mu_l}{\sigma_l c_{al}}\right)^2 + \left(\frac{\hat{\mu}_u - \mu_u}{\sigma_u c_{au}}\right)^2 + \frac{2(\hat{\mu}_l - \mu_l)(\hat{\mu}_u - \mu_u)}{\sigma_l\sigma_u c_{al}c_{au}} + 4}} \quad (29)$$

3.3. Summary of Process Capability Indices

It may be noted that $\hat{\mu}_l$ and $\hat{\mu}_u$ are the process (or estimated) means of their respective population (or target) means μ_l and μ_u. Hence, for some $k_l, k_u \neq 0$, we can write $\hat{\mu}_l = \mu_l + k_l \sigma_l$ and $\hat{\mu}_u = \mu_u + k_u \sigma_u$. If $k_l = 0$, the lower distribution is centred, if $k_u = 0$, the upper distribution is centred and if $k_l, k_u = 0$, then both are centred. Therefore, the expressions for C_p, C_{pk}, C_{pm}, C_{pkl}, C_{pku}, C_{pml} and C_{pmu} can be rewritten and summarized as follows:

$$C_p = \frac{1}{6}\left(\frac{k_{l2}}{c_{al}} + \frac{k_{u2}}{c_{au}}\right) \tag{30}$$

$$C_{pk} = min\left\{\frac{1}{6}\left(\frac{k_l}{c_{al}} + \frac{k_u}{c_{au}}\right) + \frac{k_{l2}}{3c_{al}},\quad -\frac{1}{6}\left(\frac{k_l}{c_{al}} + \frac{k_u}{c_{au}}\right) + \frac{k_{u2}}{3c_{au}}\right\} \tag{31}$$

$$C_{pm} = \frac{k_{l2} / c_{al} + k_{u2} / c_{au}}{3\sqrt{\left(\frac{k_l}{c_{al}}\right)^2 + \left(\frac{k_u}{c_{au}}\right)^2 + \frac{2(k_l)(k_u)}{c_{al}c_{au}} + 4}} \tag{32}$$

$$C_{pmk} = min\left\{\frac{2k_{l2} / c_{al}}{3\sqrt{\left(\frac{k_l}{c_{al}}\right)^2 + \left(\frac{k_u}{c_{au}}\right)^2 + \frac{2(k_l)(k_u)}{c_{al}c_{au}} + 4}},\right.$$

$$\left.\frac{2k_{u2} / c_{au}}{3\sqrt{\left(\frac{k_l}{c_{al}}\right)^2 + \left(\frac{k_u}{c_{au}}\right)^2 + \frac{2(k_l)(k_u)}{c_{al}c_{au}} + 4}}\right\} \tag{33}$$

$$C_{pkl} = \frac{1}{6}\left(\frac{k_l}{c_{al}} + \frac{k_u}{c_{au}}\right) + \frac{k_{l2}}{3c_{al}} \tag{34}$$

$$C_{pku} = -\frac{1}{6}\left(\frac{k_l}{c_{al}} + \frac{k_u}{c_{au}}\right) + \frac{k_{u2}}{3c_{au}} \tag{35}$$

$$C_{pml} = \frac{2k_{l2} / c_{al}}{3\sqrt{\left(\frac{k_l}{c_{al}}\right)^2 + \left(\frac{k_u}{c_{au}}\right)^2 + \frac{2(k_l)(k_u)}{c_{al}c_{au}} + 4}} \tag{36}$$

$$C_{pmu} = \frac{2k_{u2} / c_{au}}{3\sqrt{\left(\frac{k_l}{c_{al}}\right)^2 + \left(\frac{k_u}{c_{au}}\right)^2 + \frac{2(k_l)(k_u)}{c_{al}c_{au}} + 4}} \tag{37}$$

4. REAL TIME APPLICATION

Billard and Diday (2000) [31] considered a set of data on systolic pressure and diastolic pressure to study their influence on the pulse rate. We have used an abridged data on systolic pressure and diastolic pressure as given in Table **1** to study the PCIs proposed in our work. In this regard, we have collected the specification details for systolic diastolic values as well from internet sources. It may be noted that under normal conditions, while the systolic blood pressure is ranging between 90 to 120 millimeters of mercury (mm Hg), the diastolic blood pressure assumes values between 60 to 80 mm Hg. However these values may differ slightly depending on othere factors such as age, gender, and general health. While we assume these specification values for our application purpose, it is recommended to consult a healthcare provider for individual advice and blood pressure measurement interpretations.

Table 1: Systolic Pressure and Diastolic Pressure Data (Abridged from Billard and Diday, 2000)

Systolic Pressure		Diastolic Pressure	
X_l	X_u	X_l	X_u
90	100	50	70
90	130	70	90
140	180	90	100
110	142	80	108
90	100	50	70
130	160	80	110
60	100	140	150
130	160	76	90
110	190	70	110
138	180	90	110
110	150	78	100

While, we did not check if the data is under statistical control, our focus is to explore how the proposed PCIs can be computed and interpreted. We also intend to discuss the appropriateness of the proposed approach. We noticed that the parameters involved in the expressions of the PCIs include the shift values k_l, k_u, the inflation coefficients c_{al}, c_{au} and, in addition, we have other constants k_{l2}, k_{u2} (k_{l1}, k_{u1} do

Table 2: Computations of Constants in the Expressions of PCIs

From Specifications	Estimated Means	Standard deviations	Additional Computations
$LSL = LSL_l = 60$			$k_l = 0.929$
$USL_l = 80$			$k_u = 0.817$
$LSL_u = 90$			$k_{l1} = 0.809$
$USL = USL_u = 120$			$k_{u1} = 0.655$
$\mu_l = 70.00$	$\hat{\mu}_l = 90.09$	$\hat{\sigma}_l = 21.61$	$k_{l2} = 1.272$
$\mu_u = 105.00$	$\hat{\mu}_u = 126.81$	$\hat{\sigma}_u = 26.70$	$k_{u2} = 1.217$
$\mu_a = 87.50$	$\hat{\mu}_a = 108.45$	$\hat{\sigma}_a = 17.11$	$C_{al} = 0.792$
			$C_{au} = 0.641$

not appear in the expressions of PCIs since there are not linked to either LCL or UCL). Note that if $k_l, k_u = 0$, then there are no shifts in the targets of both lower and upper values. The values of the constants involved in the expressions of the PCIs given in the Eqns. (30) through (36) are computed and given in Table **2**.

Table 3: Computations Various PCIs

PCIs	Estimated Values	Remarks
C_p	0.584	
C_{pk}	0.225	Min{0.944, 0.225}
C_{pm}	0.370	
C_{pmk}	0.338	Min{0.338, 0.400}
C_{pkl}	0.944	
C_{pku}	0.225	
C_{pml}	0.339	
C_{pmu}	0.400	

We have now evaluated all the PCIs with the help of the expressions given in Eqns. (30) through (36) and the estimated values are provided in Table **3**. In addition, we have summarized the required values of PCIs for different sigma values in Table **4**. In fact, in Total Quality Management (TQM) arena, the processes/industries are classified as world class or industry average or noncompetitive based on the sigma quality level that is achieved [34, 35]. It may be noted that higher the sigma value reflects lesser process standard deviation and hence the process becomes more capable. For the data considered here, we observe that the PCI values are not desirable since there is a huge shift in the process means along with higher standard deviations. This reflects the situation that the specimens considered are not maintaining the healthy conditions with regard to systolic and diastolic.

5. DISCUSSIONS AND CONCLUSIONS

In this paper, we proposed to determine PCIs when the data available are interval-valued. We have specified the situations from various fields such as industries, healthcare and other service sectors where data are available in the form of intervals rather than real data sets. Initially, we have discussed most of the available PCIs that are commonly used. This includes the discussion on both one-sided and two-sided PCIs as well. Later, we have attempted to build connections between the lower and upper specifications limits based on which the resulting PCIs are drawn. This is done by considering the coefficients of inflation and the mean shift values of distributions of both lower and upper values of the interval-valued data. The new expressions for the proposed PCIs are determined. We have considered the systolic and diastolic data to demonstrate the computations of PCIs.

While every product or process quality characteristics has specification requirement, it is upto the quality practioners to maintain the process specification limits and make the product/process capable. In order to quantify the capability, PCIs of different types are used. While some are based on deviation from target, some use the variations and the specification limits (LSL and USL) to quantify the PCIs. Our goal is to enhance the assessment of process capability for processes with interval-valued data. By utilizing the concepts such as inflation coefficients, mean shifts in addition to the features of mixture probability distribution functions, the new process

Table 4: Sigma Levels and the Required PCI Values for a Capable Process of PCIs

State of the organization	Sigma Level	Minimum PCI values required for a capable process	
		Centered Process (C_p)	Shifted Process (C_{pk}, C_{pm}, C_{pmk}, C_{pkl}, C_{pku}, C_{pml}, C_{pmu})
Non-competitive	1	<1.00	<1.000
	2		
Industry average	3	1.00	0.500
	4	1.33	0.833
	5	1.67	1.167
World Class	6	>2.00	>1.500

capability indices are derived to provide an in-depth understanding of process performance. These indices give a refined approach to evaluating processes where data is inherently interval-valued, thereby improving accuracy in assessing capability and helping in effective decision-making for process improvement initiatives. This research contributes towards advancing quality control methodologies and has the potential to help healthcare settings and applications where interval-valued data is prevalent, all by bridging the gap in capability analysis for such processes.

The very focus of this research work is to propose an easily understandable approach for determining the PCIs when the data are interval-values. As discussed earlier, unlike the case where the data are real-valued and with individual observations, when the observations are interval-valued, the approach to compute PCIs is not straightforward. There is a possibility of different levels of variations with regard to either lower or higher value in the interval. The expressions given in this paper consider the flexibility to accommodate the variations in both the values of the interval to get the true inherent capability level.

While we do not have any specific limitations for the use of the proposed approach, we suggest the practitioners to consider checking the nature of statistical control of the process with respect to the quality characteristics used such systolic and diastolic variables used in this study. We refer the readers to Revichandran *et al.* (2024) [28] for one such a study on the development of control chart for interval-valued data. as a future work, we would like to numerically study the influences of various constants included in the expressions of PCIs. Also, examples of different nature pertaining to healthcare with interval-valued data will also be taken to study the feasibilities of using the proposed PCIs.

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