A "Golden Age" of the Companies: Conditions of Its Existence

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Abstract: A few years ago we have discovered the effect of the "golden age" of company (Brusov *et al.* 2015): it was shown for the first time that valuation of the weighted average cost of capital, WACC, in the Modigliani – Miller theory (Modigliani *et al.* 1958; 1963; 1966) is not minimal and valuation of the company capitalization is not maximal, as all financiers supposed up to this discovery: at some age of the company its WACC value turns out to be lower, than in Modigliani – Miller theory and company capitalization V turns out to be greater, than V in Modigliani – Miller theory. It was shown that, from the point of view of cost of attracting capital there are two types of dependences of weighted average cost of capital, WACC, on the company age n: monotonic descending with n and descending with passage through minimum, followed by a limited growth. In practice there are companies with both types of dependences of WACC on the company age n.

In this paper we investigate which companies have the "golden age", i.e. obey the latter type of dependence of WACC on n. With this aim we study the dependence of WACC on the age of company n at various leverage levels within wide spectrum of capital costs values as well as the dependence of WACC on leverage level L at fixed company age n. All calculations have been done within modern theory of capital cost and capital structure BFO by Brusov–Filatova–Orekhova (Brusov *et al.* 2011a,b,c,d,e; 2012 a,b; 2013 a,b,c; 2014 a,b; Filatova *et al.* 2008).

We have shown that existence of the "golden age" of company does not depend on the value of capital costs of the company, but depends on the difference between equity k_0 and debt k_d costs. The "golden age" of company exists at small enough difference between k_0 and k_d costs, while at high value of this difference the "golden age" of company is absent: curve WACC(n) monotonic descends with n. For the companies with the "golden age" curve WACC(L) for perpetuity companies lies between curves WACC(L) for company ages n=1 and n=3, while for the companies without the "golden age" curve WACC(L) for perpetuity companies is the lowest one.

In previous paper (Brusov *et al.* 2015) we have found also a third type of WACC(n) dependence: descending with passage through minimum, which lies below the perpetuity limit value, then going through maximum followed by a limited descending. We called this effect "Kulik effect". In this paper we have found a variety of "Kulik effect": descending with passage through minimum of WACC, which lies above the perpetuity limit value, then going through maximum followed by a limited descending. We call this company age, where WACC has a minimum, which lies above the perpetuity limit value, "a silver age" of the company.

Because the cost of attracting capital is used in rating methodologies as discounting rate under discounting of cash flows, study of WACC behavior is very important for rating procedures. The account of effects of the "golden (silver) age" could change the valuation of creditworthiness of issuers.

Remind that, since the "golden age" of company depends on the company's capital costs, by controlling them (for example, by modifying the value of dividend payments, that reflect the equity cost), company may extend the "golden age" of the company, when the cost to attract capital becomes a minimal (less than perpetuity limit), and capitalization of companies becomes maximal (above than perpetuity assessment) up to a specified time interval. We discuss the use of opened effects in developing economics.

Keywords: Brusov–Filatova–Orekhova theory, Modigliani – Miller theory, minimal capital cost of company, the "golden age" of the company, rating methodology.

1. INTRODUCTION

In this paper we answer the following question: which companies have "a golden age", i.e. obey the following type of dependence of WACC on n: WACC(n) descending with passage through minimum, followed by a limited growth. With this aim we study the dependence of WACC on the age of company n at various leverage levels within wide spectrum of capital costs values as well as the dependence of WACC on leverage level L at fixed company age n. All calculations have been done within modern theory of capital cost and capital structure BFO by Brusov–Filatova–Orekhova (Brusov *et al.* 2011a,b,c,d,e; 2012

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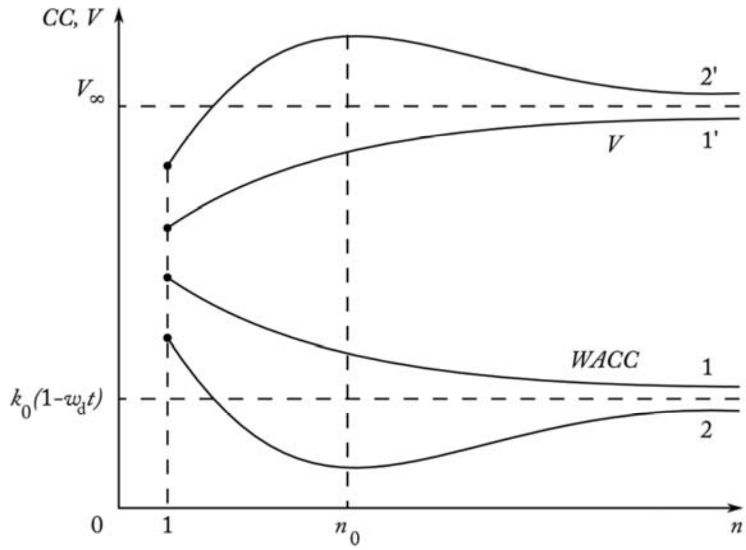

Figure 1: Two kind of dependences of weighted average cost of capital, WACC, and company capitalization, V, on life-time of the company *n*:

a,b; 2013 a,b,c; 2014 a,b; Filatova *et al.* 2008). We make calculations for equity cost k_0 (at L=0) between 6% and 30% and debt cost k_d between 4% and 28% for a lot of pairs (k_0, k_d), accounting that the inequality $k_0 \geq k_d$ is always valid via the fact that equity cost is more risky than debt one. We present in paper only some examples of our calculations (one-two in each group) and readers should understand that other results in each group give more or less qualitatively similar results.

We have shown that existence of the "golden age" of company depends not on the value of capital costs of the company, but on the difference between equity k_0 and debt k_d costs. The "golden age" of company exists at small enough difference between k_0 and k_d costs, while at high value of this difference the "golden age" of company is absent: curve WACC(n) monotonic descends with n. For the companies with the "golden age" curve WACC(L) for perpetuity limit (n=∞) lies between curves WACC(L) for one year (n=1) and three years (n=3) companies, while for the companies without the "golden age" curve WACC(L) for perpetuity limit is the lowest one.

The problem of the existence of the "golden age" of company is very important in ratings because the discount rate (WACC value), used in discounting of cash flows in ratings, depends on the existence or the nonexistence of the "golden age" of company.

1-1'- monotonic dependence of weighted average cost of capital, WACC, and company capitalization, V, on life-time of the company *n*;

2-2'- showing descending of WACC with n, and with the passage through a minimum and then a limited growth and increase of V with the passage through a maximum (at n_0) and then a limited descending.

2. COMPANIES WITHOUT THE "GOLDEN AGE" (LARGE DIFFERENCE BETWEEN k_0 AND k_d COSTS)

As an example of companies without the "golden age"(with large difference between k_0 and k_d costs) we present the calculations for equity cost k_0 (at L=0) equals to 20% and debt cost k_d equals to 9%.

2.1. Dependence of Weighted Average Cost of Capital, WACC, on the Company Age *n* at Different Leverage Levels

We study below the dependence of weighted average cost of capital, WACC, on the company age *n* at different leverage levels (L= 1,2, 3), using the BFO formula

$$\frac{\left[1-(1+WACC)^{-n}\right]}{WACC}=\frac{\left[1-(1+k_0)^{-n}\right]}{k_0\left[1-\omega_d t\left(1-(1+k_d)^{-n}\right)\right]} \quad (1)$$

Leverage level *L* presents in BFO formula through the share of debt capital $w_d = L/(1+L)$.

The results of our calculations are shown below in Tables and Figures.

It is seen from Tables **1-3** and Figure **2** that 1-1' - behavior (from Figure **1**) takes place: monotonic

For L=1 one has

Table 1:

n	t	L	WACC(L=1)	ko	kd	wd	A(n)	БФО
1	0,2	1	0,19001348	0,2	0,09	0,50	0,840271	0,000055264
2	0,2	1	0,18679105	0,2	0,09	0,50	1,552355	0,000242408
3	0,2	1	0,18521681	0,2	0,09	0,50	2,155589	0,000643588
4	0,2	1	0,18453895	0,2	0,09	0,50	2,666482	0,000008562
5	0,2	1	0,18398934	0,2	0,09	0,50	3,099102	0,000019534
6	0,2	1	0,18361179	0,2	0,09	0,50	3,465420	0,000038686
7	0,2	1	0,18333561	0,2	0,09	0,50	3,775614	0,000068644
8	0,2	1	0,18312210	0,2	0,09	0,50	4,038322	0,000111902
9	0,2	1	0,18294855	0,2	0,09	0,50	4,260871	0,000170582
10	0,2	1	0,18280097	0,2	0,09	0,50	4,449469	0,000246231
20	0,2	1	0,18181178	0,2	0,09	0,50	5,305460	0,000013165
30	0,2	1	0,18103559	0,2	0,09	0,50	5,486207	0,000041589
40	0,2	1	0,18052092	0,2	0,09	0,50	5,532205	0,000065502

For L=2 we have

Table 2:

n	t	L	WACC(L=2)	ko	kd	wd	A(n)	БФО
1	0,2	2	0,18670845	0,2	0,09	0,67	0,8426098	0,000057189
2	0,2	2	0,18242414	0,2	0,09	0,67	1,5607236	0,000239282
3	0,2	2	0,18033977	0,2	0,09	0,67	2,1724715	0,000618675
4	0,2	2	0,17935211	0,2	0,09	0,67	2,6934467	0,000007915
5	0,2	2	0,17860838	0,2	0,09	0,67	3,1370358	0,000018207
6	0,2	2	0,17809570	0,2	0,09	0,67	3,5147106	0,000036130
7	0,2	2	0,17771992	0,2	0,09	0,67	3,8362859	0,000064261
8	0,2	2	0,17742943	0,2	0,09	0,67	4,1101468	0,000105021
9	0,2	2	0,17719390	0,2	0,09	0,67	4,3434469	0,000160484
10	0,2	2	0,17699457	0,2	0,09	0,67	4,5422818	0,000232186
20	0,2	2	0,17567694	0,2	0,09	0,67	5,4686271	0,000012229
30	0,2	2	0,17467722	0,2	0,09	0,67	5,6790746	0,000039727
40	0,2	2	0,17401430	0,2	0,09	0,67	5,7372042	0,000064124

For L=3 one has

Table 3:

n	t	L	WACC(L=3)	ko	kd	wd	A(n)	БФО
1	0,2	3	0,18506498	0,2	0,09	0,75	0,8437839	0,000051714
2	0,2	3	0,18024369	0,2	0,09	0,75	1,5649420	0,000228517
3	0,2	3	0,17789664	0,2	0,09	0,75	2,1810121	0,000607678
4	0,2	3	0,17675215	0,2	0,09	0,75	2,7071344	0,000007660

(Table 3). Continued.

n	t	L	WACC(L=3)	ko	kd	wd	A(n)	БФО
5	0,2	3	0,17590910	0,2	0,09	0,75	3,1563532	0,000017650
6	0,2	3	0,17532683	0,2	0,09	0,75	3,5398852	0,000035095
7	0,2	3	0,17489950	0,2	0,09	0,75	3,8673589	0,000062558
8	0,2	3	0,17456914	0,2	0,09	0,75	4,1470258	0,000101532
9	0,2	3	0,17430145	0,2	0,09	0,75	4,3859468	0,000155351
10	0,2	3	0,17407533	0,2	0,09	0,75	4,5901557	0,000225050
20	0,2	3	0,17259416	0,2	0,09	0,75	5,5540330	0,000011720
30	0,2	3	0,17148952	0,2	0,09	0,75	5,7806846	0,000038586
40	0,2	3	0,17075742	0,2	0,09	0,75	5,8455083	0,000063263

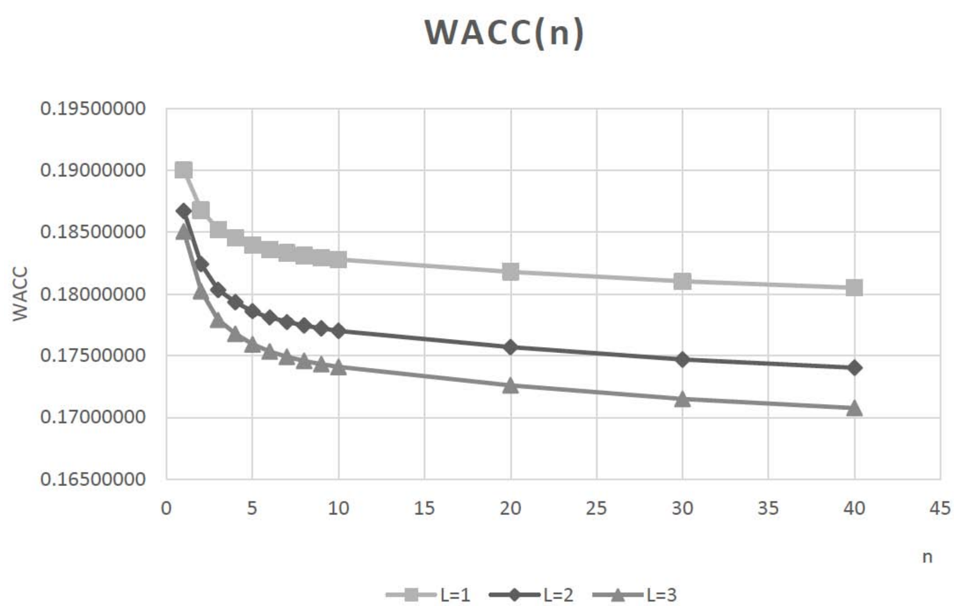

Figure 2: The dependence of weighted average cost of capital, WACC, on the company age *n* at different leverage levels (L= 1,2, 3).

Table 4:

n	t	L	WACC(n=1)	ko	kd	wd	A(n)	БФО
1	0,2	0	0,19990871	0,2	0,09	0,00	0,833333	0,000063399
1	0,2	1	0,18989673	0,2	0,09	0,50	0,840271	0,000137715
1	0,2	2	0,18675471	0,2	0,09	0,67	0,842610	0,000024342
1	0,2	3	0,18504185	0,2	0,09	0,75	0,843784	0,000068187
1	0,2	4	0,18395964	0,2	0,09	0,80	0,844490	0,000133490
1	0,2	5	0,18323424	0,2	0,09	0,83	0,844961	0,000179952
1	0,2	6	0,18270822	0,2	0,09	0,86	0,845298	0,000218846
1	0,2	7	0,18230907	0,2	0,09	0,88	0,845551	0,000251369
1	0,2	8	0,18203277	0,2	0,09	0,89	0,845748	0,000252248
1	0,2	9	0,18178544	0,2	0,09	0,90	0,845906	0,000271780
1	0,2	10	0,18158213	0,2	0,09	0,91	0,846034	0,000288455

Table 5:

n	t	L	WACC (n=3)	ko	kd	wd	A(n)	БФО
3	0,2	0	0,19978349	0,2	0,09	0,00	2,106481	0,000714376
3	0,2	1	0,18521867	0,2	0,09	0,50	2,155589	0,000637220
3	0,2	2	0,18034194	0,2	0,09	0,67	2,172471	0,000611094
3	0,2	3	0,17789941	0,2	0,09	0,75	2,181012	0,000597961
3	0,2	4	0,17643254	0,2	0,09	0,80	2,186169	0,000590059
3	0,2	5	0,17545407	0,2	0,09	0,83	2,189620	0,000584783
3	0,2	6	0,17475488	0,2	0,09	0,86	2,192092	0,000581010
3	0,2	7	0,17423034	0,2	0,09	0,88	2,193950	0,000578178
3	0,2	8	0,17382227	0,2	0,09	0,89	2,195397	0,000575974
3	0,2	9	0,17349576	0,2	0,09	0,90	2,196556	0,000574210
3	0,2	10	0,17322858	0,2	0,09	0,91	2,197505	0,000572766

Table 6:

n	t	L	WACC(n=45)	ko	kd	wd	A(n)	БФО
45	0,2	0	0,19999699	0,2	0,09	0,00	4,998633	0,000075004
45	0,2	1	0,18035710	0,2	0,09	0,50	5,541296	0,000073385
45	0,2	2	0,17380207	0,2	0,09	0,67	5,749351	0,000072648
45	0,2	3	0,17052242	0,2	0,09	0,75	5,859349	0,000072233
45	0,2	4	0,16855385	0,2	0,09	0,80	5,927391	0,000071966
45	0,2	5	0,16724112	0,2	0,09	0,83	5,973638	0,000071780
45	0,2	6	0,16630328	0,2	0,09	0,86	6,007115	0,000071642
45	0,2	7	0,16559980	0,2	0,09	0,88	6,032471	0,000071536
45	0,2	8	0,16505258	0,2	0,09	0,89	6,052340	0,000071451
45	0,2	9	0,16461477	0,2	0,09	0,90	6,068330	0,000071382
45	0,2	10	0,16425654	0,2	0,09	0,91	6,081476	0,000071325

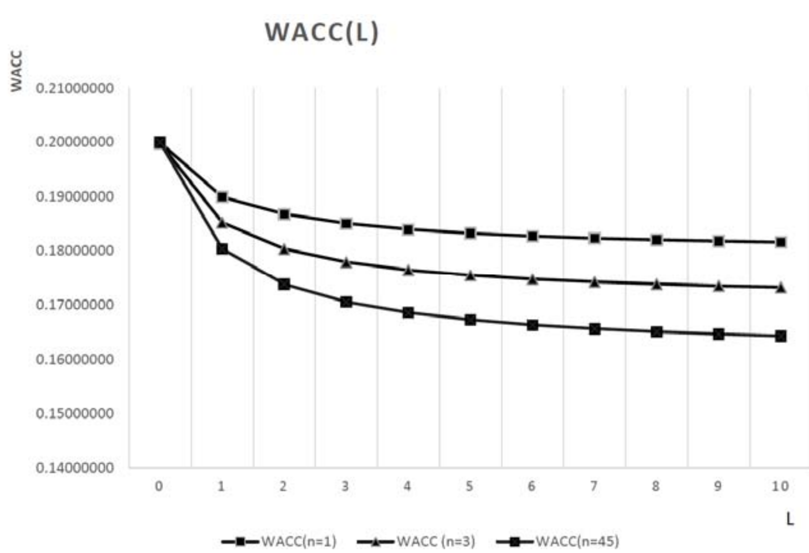

Figure 3: The dependence of weighted average cost of capital, WACC, on the leverage level at different company age *n* (n=1,3,45).

Table 7:

n	t	L	WACC	K_0	K_d	W_d	A(n)	
1	0,2	1	0,2441	0,27	0,25	0,50	0,8035	0,000292
2	0,2	1	0,2381	0,27	0,25	0,50	1,4600	0,000038
3	0,2	1	0,2360	0,27	0,25	0,50	1,9928	0,000286
4	0,2	1	0,2357	0,27	0,25	0,50	2,4231	0,000046
5	0,2	1	0,2359	0,27	0,25	0,50	2,7688	0,000319
6	0,2	1	0,2363	0,27	0,25	0,50	3,0457	0,000877
7	0,2	1	0,2371	0,27	0,25	0,50	3,2668	0,000033
8	0,2	1	0,2377	0,27	0,25	0,50	3,4430	0,000070
9	0,2	1	0,2383	0,27	0,25	0,50	3,5830	0,000127
10	0,2	1	0,2389	0,27	0,25	0,50	3,6941	0,000204
20	0,2	1	0,2422	0,27	0,25	0,50	4,0755	0,000005
30	0,2	1	0,2429	0,27	0,25	0,50	4,1115	0,000009
40	0,2	1	0,2430	0,27	0,25	0,50	4,1149	0,000010

Table 8:

n	t	L	WACC	K_0	K_d	W_d	A(n)	
1	0,2	2	0,2361	0,27	0,25	0,67	0,8090	0,000024
2	0,2	2	0,2274	0,27	0,25	0,67	1,4784	0,000164
3	0,2	2	0,2246	0,27	0,25	0,67	2,0275	0,000411
4	0,2	2	0,2239	0,27	0,25	0,67	2,4748	0,000753
5	0,2	2	0,2244	0,27	0,25	0,67	2,8370	-0,000018
6	0,2	2	0,2251	0,27	0,25	0,67	3,1288	-0,000026
7	0,2	2	0,2259	0,27	0,25	0,67	3,3630	-0,000037
8	0,2	2	0,2267	0,27	0,25	0,67	3,5504	-0,000046
9	0,2	2	0,2276	0,27	0,25	0,67	3,6999	-0,000054
10	0,2	2	0,2284	0,27	0,25	0,67	3,8189	-0,000063
20	0,2	2	0,2328	0,27	0,25	0,67	4,2301	-0,000110
30	0,2	2	0,2338	0,27	0,25	0,67	4,2694	-0,000124
40	0,2	2	0,2340	0,27	0,25	0,67	4,2731	-0,000128

Table 9:

n	t	L	WACC	K_0	K_d	W_d	A(n)	
1	0,2	3	0,2318	0,27	0,25	0,75	0,8118	0,000064
2	0,2	3	0,2219	0,27	0,25	0,75	1,4877	0,000412
3	0,2	3	0,2190	0,27	0,25	0,75	2,0453	-0,000020
4	0,2	3	0,2183	0,27	0,25	0,75	2,5015	-0,000044
5	0,2	3	0,2186	0,27	0,25	0,75	2,8723	-0,000076
6	0,2	3	0,2193	0,27	0,25	0,75	3,1721	-0,000112
7	0,2	3	0,2203	0,27	0,25	0,75	3,4133	-0,000154
8	0,2	3	0,2212	0,27	0,25	0,75	3,6067	-0,000195
9	0,2	3	0,2222	0,27	0,25	0,75	3,7612	-0,000234
10	0,2	3	0,2231	0,27	0,25	0,75	3,8845	-0,000271
20	0,2	3	0,2281	0,27	0,25	0,75	4,3120	-0,000497
30	0,2	3	0,2293	0,27	0,25	0,75	4,3530	-0,000570
40	0,2	3	0,2295	0,27	0,25	0,75	4,3569	-0,000590

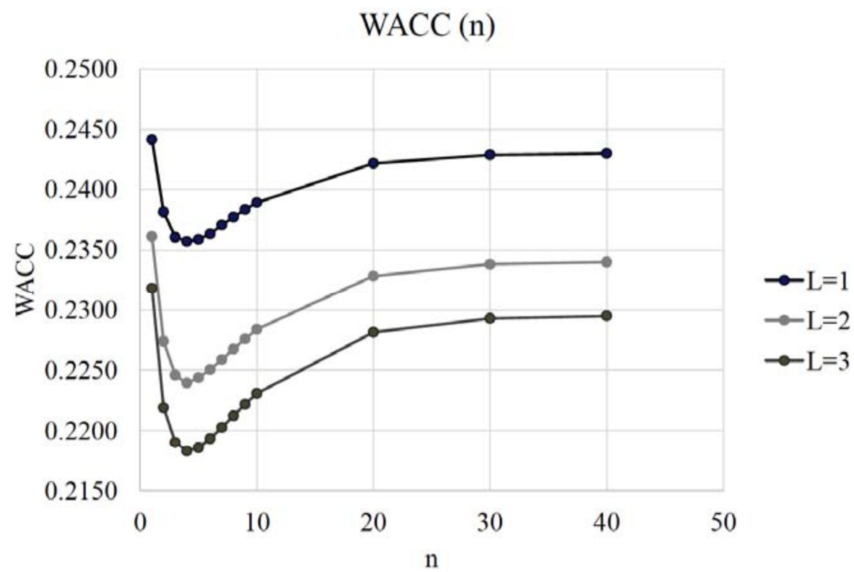

Figure 4: The dependence of weighted average cost of capital, WACC, on the company age *n* at different leverage levels (L= 1,2, 3).

Table 10:

n	t	L	WACC	K_0	K_d	W_d	A(n)	
1	0,2	0	0,2697	0,27	0,25	0,00	0,7874	0,000188
1	0,2	1	0,2441	0,27	0,25	0,50	0,8035	0,000292
1	0,2	2	0,2360	0,27	0,25	0,67	0,8090	0,000064
1	0,2	3	0,2317	0,27	0,25	0,75	0,8118	0,000153
1	0,2	4	0,2290	0,27	0,25	0,80	0,8134	0,000216
1	0,2	5	0,2273	0,27	0,25	0,83	0,8146	0,000263
1	0,2	6	0,2260	0,27	0,25	0,86	0,8154	0,000298
1	0,2	7	0,2251	0,27	0,25	0,88	0,8160	0,000326
1	0,2	8	0,2243	0,27	0,25	0,89	0,8164	0,000349
1	0,2	9	0,2237	0,27	0,25	0,90	0,8168	0,000367
1	0,2	10	0,2232	0,27	0,25	0,91	0,8171	0,000382

Table 11:

n	t	L	WACC	K_0	K_d	W_d	A(n)	
3	0,2	0	0,2697	0,27	0,25	0,00	1,8956	0,000832
3	0,2	1	0,2361	0,27	0,25	0,50	1,9928	0,000174
3	0,2	2	0,2246	0,27	0,25	0,67	2,0275	0,000411
3	0,2	3	0,2188	0,27	0,25	0,75	2,0453	0,000542
3	0,2	4	0,2154	0,27	0,25	0,80	2,0561	0,000631
3	0,2	5	0,2131	0,27	0,25	0,83	2,0634	0,000694
3	0,2	6	0,2114	0,27	0,25	0,86	2,0687	0,000741
3	0,2	7	0,2102	0,27	0,25	0,88	2,0726	0,000778
3	0,2	8	0,2092	0,27	0,25	0,89	2,0757	0,000807
3	0,2	9	0,2084	0,27	0,25	0,90	2,0781	0,000830
3	0,2	10	0,2078	0,27	0,25	0,91	2,0802	0,000850

Table 12:

n	t	L	WACC	K_0	K_d	W_d	A(n)	
49	0,2	0	0,2700	0,27	0,25	0,00	3,7037	-0,000037
49	0,2	1	0,2430	0,27	0,25	0,50	4,1152	0,000041
49	0,2	2	0,2340	0,27	0,25	0,67	4,2735	0,000159
49	0,2	3	0,2295	0,27	0,25	0,75	4,3572	0,000273
49	0,2	4	0,2268	0,27	0,25	0,80	4,4091	0,000368
49	0,2	5	0,2250	0,27	0,25	0,83	4,4444	0,000444
49	0,2	6	0,2237	0,27	0,25	0,86	4,4699	0,000505
49	0,2	7	0,2227	0,27	0,25	0,88	4,4893	0,000556
49	0,2	8	0,2220	0,27	0,25	0,89	4,5045	0,000598
49	0,2	9	0,2214	0,27	0,25	0,90	4,5167	0,000633
49	0,2	10	0,2209	0,27	0,25	0,91	4,5267	0,000664

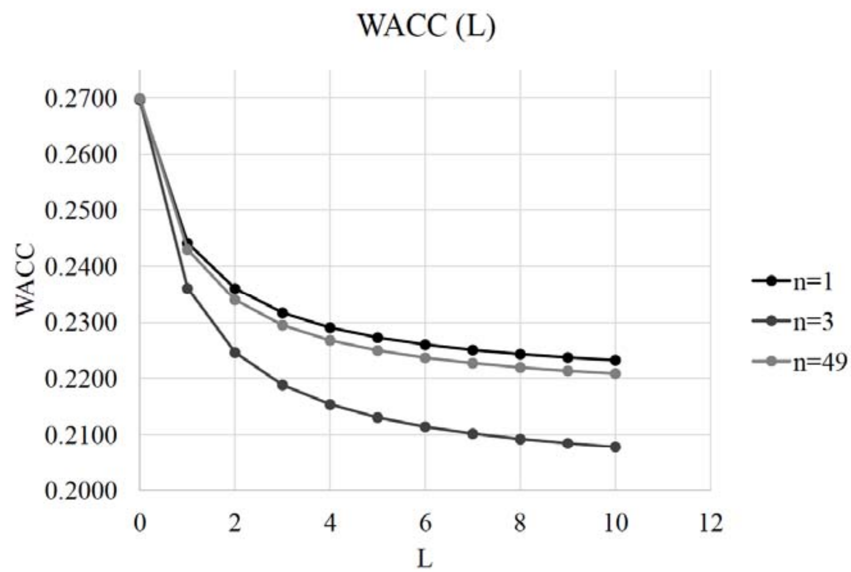

Figure 5: The dependence of weighted average cost of capital, WACC, on leverage level L at different the company age *n* (*n*= 1,3, 49).

dependence of weighted average cost of capital, WACC, and company capitalization, V, on the company age *n* for all considered leverage levels (L=1,2,3); this means that the "golden age" of company is absent. The ordering of curves is the following: the lower curve corresponds the greater leverage level.

From Tables **4-6** and Figure **3** it is seen that the ordering of curves WACC(L) is the following: the lower curve corresponds the greater company age n. We will see below that under existence the "golden age" of company this ordering will be a different one. We keep here the case of n=45 as the case which is closed to perpetuity limit. An alternative method is the using of Modigliani – Miller formula

$$WACC = k_0\left(1 - \omega_d t\right) \tag{2}$$

which follows from BFO formula (1) for perpetuity limit.

3. COMPANIES WITH THE "GOLDEN AGE" (SMALL DIFFERENCE BETWEEN k_0 AND k_d COSTS)

As an example of companies with the "golden age"(with small difference between k_0 and k_d costs) we present the calculations for equity cost k_0 (at L=0) equals to 27% and debt cost k_d equals to 25%.

It is seen from Tables **7-9** and Figure **5** that 2-2' - behavior (from Figure **1**) takes place: descending of WACC with n, and with the passage through a minimum and then a limited growth and increase of V with the passage through a maximum (at $n_0 \approx 4$) and then a limited descending. This means the presence of the "golden age" of company. The ordering of curves is the following: the lower curve corresponds the greater leverage level.

From Tables **10-12** and Figure **6** the quite new effect follows: the ordering of curves WACC(L) is the following: the top curve corresponds to the company age n=1, the middle one corresponds to perpetuity company $n_0 = \infty$ (we use n=49 to approximate perpetuity limit), and bottom one corresponds to the company age n=3. Thus, the curve WACC(L) for perpetuity company lies between curves corresponding to the company age n=1, and n-3.

Note, that this ordering is quite different from the case when the "golden age" of company is absent: in that case the lower curve corresponds the greater company age n: the top curve corresponds to the company age n=1, the middle one corresponds to the company age n=3 and bottom one corresponds to the perpetuity company.

4. COMPANIES WITH ABNORMAL "GOLDEN AGE" (INTERMEDIATE DIFFERENCE BETWEEN k_0 AND k_d COSTS)

One example, which is different from two considered above cases will be studied below, where we present the calculations for equity cost k_0 (at L=0) equals to 27% and debt cost k_d equals to 16%.

Table 13:

n	t	L	K_0	K_d	W_d	WACC
1	0,2	1	0,27	0,16	0,5	0,252483428
2	0,2	1	0,27	0,16	0,5	0,247301552
3	0,2	1	0,27	0,16	0,5	0,245105573
4	0,2	1	0,27	0,16	0,5	0,244045922
5	0,2	1	0,27	0,16	0,5	0,243511488
6	0,2	1	0,27	0,16	0,5	0,243250247
7	0,2	1	0,27	0,16	0,5	0,24313388
8	0,2	1	0,27	0,16	0,5	0,243097298
9	0,2	1	0,27	0,16	0,5	0,243103247
10	0,2	1	0,27	0,16	0,5	0,243130071
20	0,2	1	0,27	0,16	0,5	0,243291667
30	0,2	1	0,27	0,16	0,5	0,243146312
40	0,2	1	0,27	0,16	0,5	0,243048121
∞	0,2	1	0,27	0,16	0,5	0,243

Table 14:

n	t	L	K_0	K_d	W_d	WACC
1	0,2	2	0,27	0,16	0,6(6)	0,246644883
2	0,2	2	0,27	0,16	0,6(6)	0,239710388
3	0,2	2	0,27	0,16	0,6(6)	0,236752006
4	0,2	2	0,27	0,16	0,6(6)	0,235311829
5	0,2	2	0,27	0,16	0,6(6)	0,234578628
6	0,2	2	0,27	0,16	0,6(6)	0,23421254
7	0,2	2	0,27	0,16	0,6(6)	0,234046353
8	0,2	2	0,27	0,16	0,6(6)	0,233991091
9	0,2	2	0,27	0,16	0,6(6)	0,233996344
10	0,2	2	0,27	0,16	0,6(6)	0,234032536
20	0,2	2	0,27	0,16	0,6(6)	0,234316362
30	0,2	2	0,27	0,16	0,6(6)	0,234174539
40	0,2	2	0,27	0,16	0,6(6)	0,234059889
∞	0,2	2	0,27	0,16	0,6(6)	0,234

Table 15:

n	t	L	K_0	K_d	W_d	WACC
1	0,2	3	0,27	0,16	0,75	0,243725045
2	0,2	3	0,27	0,16	0,75	0,235910011
3	0,2	3	0,27	0,16	0,75	0,232564477
4	0,2	3	0,27	0,16	0,75	0,230926014
5	0,2	3	0,27	0,16	0,75	0,230090888
6	0,2	3	0,27	0,16	0,75	0,229669501
7	0,2	3	0,27	0,16	0,75	0,229475982
8	0,2	3	0,27	0,16	0,75	0,229409857
9	0,2	3	0,27	0,16	0,75	0,229414009
10	0,2	3	0,27	0,16	0,75	0,229454135
20	0,2	3	0,27	0,16	0,75	0,229812629
30	0,2	3	0,27	0,16	0,75	0,229682914
40	0,2	3	0,27	0,16	0,75	0,229564113
∞	0,2	3	0,27	0,16	0,75	0,2295

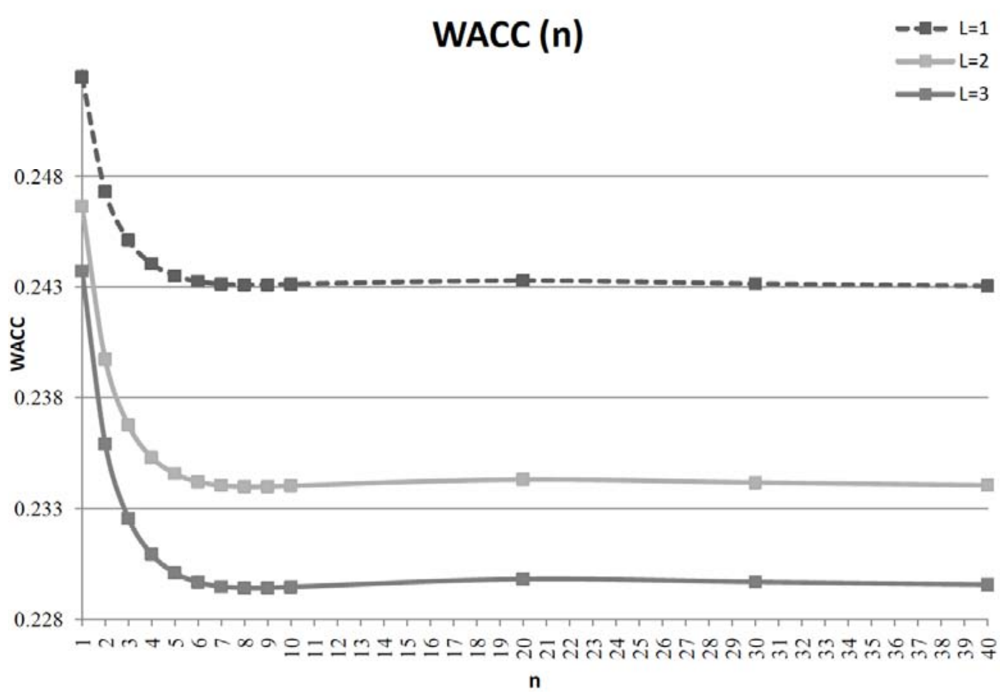

Figure 6: The dependence of weighted average cost of capital, WACC, on the company age *n* at different leverage levels (L= 1,2, 3).

While in this case the "golden age" of the company is present but it is less pronounced: the minimal WACC value (at some leverage value: in this case at L=1) lies above the perpetuity WACC value. We call this situation the "silver age" of the company.

To be sure that the minimal WACC value at leverage level L=1 lies above the perpetuity WACC value we make more detailed calculations for this case (see Table **16**). We see that the minimal WACC value at n=8,2 is equal to 0,243095889, while perpetuity limit is equal 0,243 and lies below.

Let us study the dependence of weighted average cost of capital, WACC, on the leverage level at different company age *n* (*n*= 1, 3, ∞)

Table 16:

L	K_o	K_d	t	n	W_d	WACC	BFO
1	0,27	0,16	0,2	7	0,50	0,243133854	0,000
1	0,27	0,16	0,2	7,2	0,50	0,243121633	0,000
1	0,27	0,16	0,2	7,4	0,50	0,243112185	0,000
1	0,27	0,16	0,2	7,6	0,50	0,243105150	0,000
1	0,27	0,16	0,2	7,8	0,50	0,243100256	0,000
1	0,27	0,16	0,2	8	0,50	0,243097246	0,000
1	0,27	0,16	0,2	8,2	0,50	0,243095889	0,000
1	0,27	0,16	0,2	8,4	0,50	0,243095979	0,000
1	0,27	0,16	0,2	8,6	0,50	0,243097328	0,000
1	0,27	0,16	0,2	8,8	0,50	0,243099771	0,000
1	0,27	0,16	0,2	9	0,50	0,243103156	0,000
1	0,27	0,16	0,2	∞	0,50	0,243	0,000

Table 17:

n	t	L	K_0	K_d	W_d	WACC
1	0,2	0	0,27	0,16	0	0,270000213
1	0,2	1	0,27	0,16	0,5	0,252483428
1	0,2	2	0,27	0,16	0,6(6)	0,246644883
1	0,2	3	0,27	0,16	0,75	0,243725045
1	0,2	4	0,27	0,16	0,8	0,241973361
1	0,2	5	0,27	0,16	0,8(3)	0,24080557
1	0,2	6	0,27	0,16	0,857142857	0,239971433
1	0,2	7	0,27	0,16	0,875	0,239345829
1	0,2	8	0,27	0,16	0,8(8)	0,238859248
1	0,2	9	0,27	0,16	0,9	0,238469982
1	0,2	10	0,27	0,16	0,(90)	0,238151492

Table 18:

n	t	L	K_0	K_d	W_d	WACC
3	0,2	0	0,27	0,16	0	0,270000842
3	0,2	1	0,27	0,16	0,5	0,245105573
3	0,2	2	0,27	0,16	0,6(6)	0,236752006
3	0,2	3	0,27	0,16	0,75	0,232564477
3	0,2	4	0,27	0,16	0,8	0,230048473
3	0,2	5	0,27	0,16	0,8(3)	0,228369673
3	0,2	6	0,27	0,16	0,857142857	0,22716981
3	0,2	7	0,27	0,16	0,875	0,226269517
3	0,2	8	0,27	0,16	0,(8)	0,225569054
3	0,2	9	0,27	0,16	0,9	0,225008535
3	0,2	10	0,27	0,16	0,(90)	0,224549831

Table 19:

n	t	L	K_0	K_d	W_d	WACC
∞	0,2	0	0,27	0,16	0	0,27
∞	0,2	1	0,27	0,16	0,5	0,243
∞	0,2	2	0,27	0,16	0,(6)	0,234
∞	0,2	3	0,27	0,16	0,75	0,2295
∞	0,2	4	0,27	0,16	0,8	0,2268
∞	0,2	5	0,27	0,16	0,8(3)	0,225
∞	0,2	6	0,27	0,16	0,857142857	0,223714286
∞	0,2	7	0,27	0,16	0,875	0,22275
∞	0,2	8	0,27	0,16	0,(8)	0,222
∞	0,2	9	0,27	0,16	0,9	0,2214
∞	0,2	10	0,27	0,16	0,(90)	0,220909091

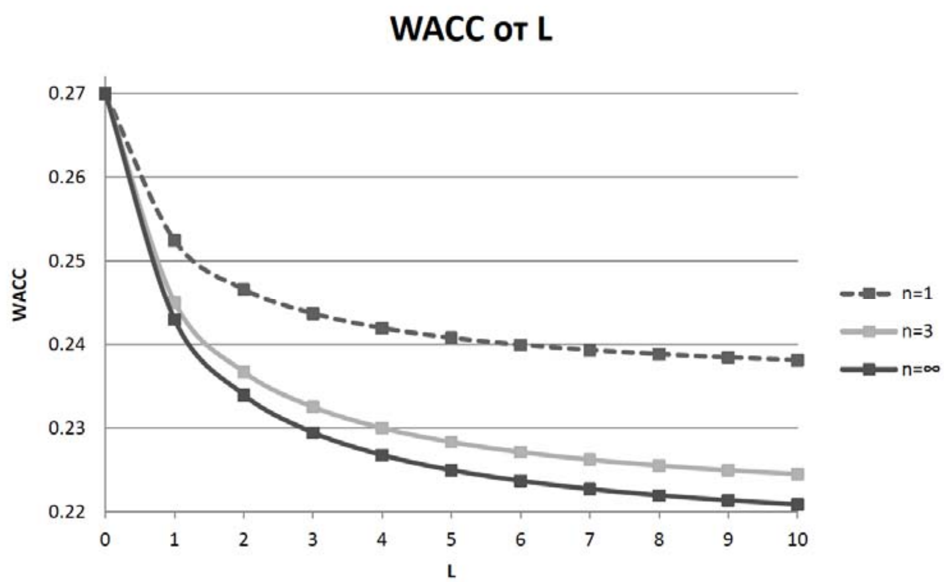

Figure 7: The dependence of weighted average cost of capital, WACC, on the leverage level at different company age *n* (n=1,3,∞).

It is seen from Tables **14-15** and Figure **6** that the following behavior takes place for L=2 and 3: a third modification of dependences of weighted average cost of capital, WACC, on the company age n takes place: descending of WACC with passage through minimum at n=8, followed by a growth with passage through maximum at n=20 and finally with trend to perpetuity limit from bigger values (remind, that at second type of WACC(n) behavior, the curve WACC(n) tends to perpetuity limit from lower values). We have called this effect "Kulik effect".

The ordering of curves is the following: the lower curve corresponds the greater leverage level.

From Tables **16-19** and Figure **6** it is seen that for L=1 the following behavior takes place: descending of WACC with passage through minimum at n=8,2 (which is higher than perpetuity limit), followed by a growth with passage through maximum at n=20 and finally with trend to perpetuity limit from bigger values. This means that the "golden age" in its purest form presents at leverage levels for L=2 and 3, while at L=1 one has different effect: we call it "silver age"

The ordering of curves is the following: the lower curve corresponds the greater company age.

It turns out that at particular values of capital costs, for example, at k_0 =27%; k_d =16%, a third modification

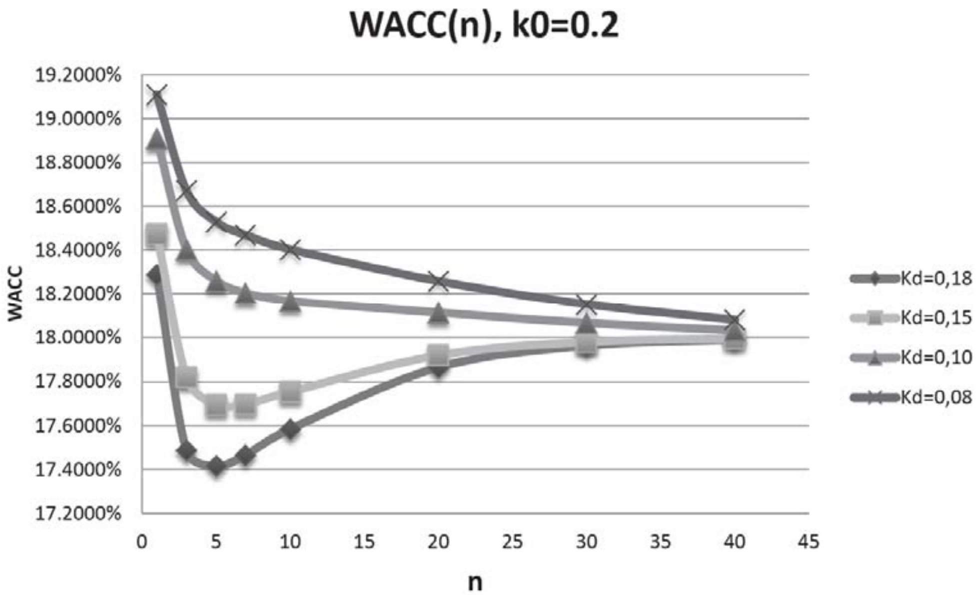

Figure 8: Dependence of weighted average cost of capital, WACC, on life-time of the company *n* at fixed value of equity cost, k_0 =20%, and at four values of debt cost, k_d =8%;10%;15% and 18% at leverage level L=1.

Table 20:

k_d	**0.18**	**0.15**	**0.10**	**0.08**
$\Delta k = k_0 - k_d$	0.02	0.05	0.10	0.12
$\Delta WACC$, %	3.38	1.89	NA	NA

of dependences of weighted average cost of capital, WACC, on the company age n takes place: descending of WACC with passage through minimum, followed by a growth with passage through maximum and finally with trend to perpetuity limit from bigger values (remind, that at second type of WACC(n) behavior, the curve WACC(n) tends to perpetuity limit from lower values). We have called this effect "Kulik effect".

5. COMPARING WITH RESULTS FROM PREVIOUS PAPER

5.1. Under Change of the Debt Capital Cost, k_d

From Figure **8** it is seen, that with increase of debt cost, k_d, the character of dependence of weighted average cost of capital, WACC, on the company age *n* is changed from monotonic descending of WACC with *n* to descending of WACC with n with passage through minimum, followed by a limited growth. It is seen from the Table **20** that the gap depth $\Delta WACC$ (the difference between the optimal (minimal) value of weighted average cost of capital, WACC, and its perpetuity limit) decreases with $\Delta k = k_0 - k_d$ from 3.38% at $\Delta k = 0.02$ up to 1.89% at $\Delta k = 0.5$. At $\Delta k = 0.10$ and $\Delta k = 0.12$ the minimum in dependence of WACC(n) is absent (too big value of $\Delta k = k_0 - k_d$). This coincides with our conclusions in this paper.

The same conclusion could be made from Figure **9** and Table **21** for higher values of capital costs: it is seen, that with increase of debt cost, k_d at fixed k_0 , i.e. with decrease $\Delta k = k_0 - k_d$ the gap depth $\Delta WACC$ is increased from 1.08% at $\Delta k = 0.10$ up to 1.85% at $\Delta k = 0.05$. This as well coincides with our conclusions in this paper.

5.2. Under Change of the Equity Capital Cost, k_0 .

From Figure **10** and Table **22** it is seen, that the gap depth $\Delta WACC$ (the difference between the optimal (minimal) value of weighted average cost of capital, WACC, and its perpetuity limit) decreases with $\Delta k = k_0 - k_d$ from 0.55% at $\Delta k = 0.01$ up to 0.03% at $\Delta k = 0.10$. This as well coincides with our conclusions in this paper.

In conclusion we present at Figure **11** the both cases of "Kulik" effect: "the golden age" of the company and "the "silver age" of the company.

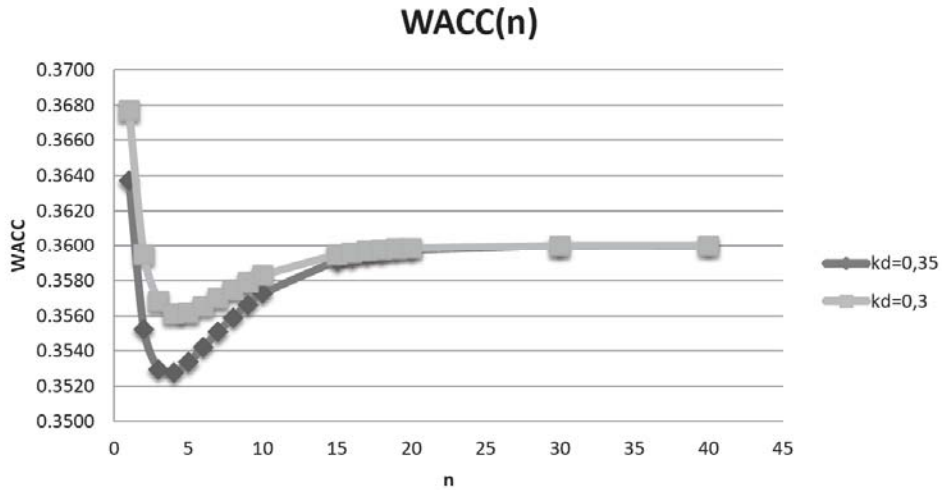

Figure 9: Dependence of weighted average cost of capital, WACC, on life-time of the company *n* at fixed high value of equity cost, k_0 =40%, and two values of debt cost, k_d =30% and 35% at leverage level L=1.

Table 21:

$k_0 = 0,4; k_d$	**0.35**	**0.3**
$\Delta k = k_0 - k_d$	0.05	0.10
$\Delta WACC$, %	1.85	1.08

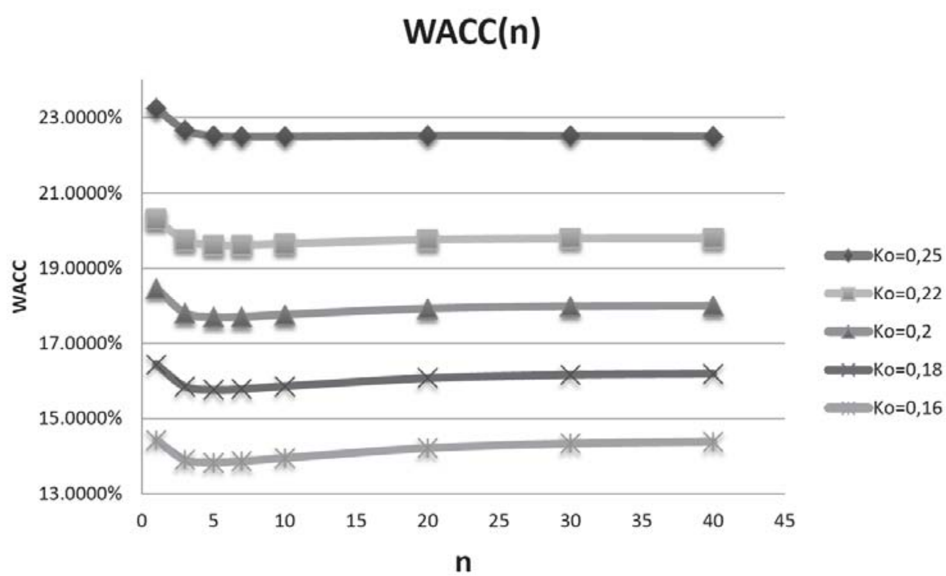

Figure 10: Dependence of weighted average cost of capital, WACC, on life-time of the company *n* at fixed value of debt cost, k_d =15%, and five values of equity cost, k_0 =16%;18%;20%;22% and 25% at leverage level L=1.

Table 22:

k_0	**0.16**	**0.18**	**0.20**	**0.22**	**0.25**
$\Delta k = k_0 - k_d$	0.01	0.03	0.05	0.07	0.10
$\Delta WACC$, %	0.55	0.43	0.30	0.18	0.03

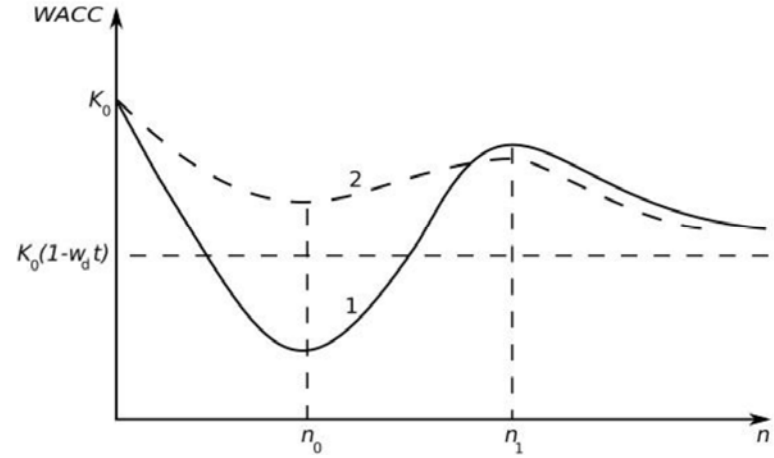

Figure 11: Dependence of weighted average cost of capital, WACC, on company age of the company *n*, which illustrate the presence of "the golden age" of the company (curve 1) and of "the silver age" of the company (curve 2) under existence of "Kulik" effect. Here n_0 is "the golden (silver) age" of the company and n_1 is the age of local maximum in dependence of WACC(n).

CONCLUSIONS

In our previous paper a few years ago (Brusov *et al*. 2015) we have discovered the effect of the "golden age" of company: it was shown for the first time that valuation of the weighted average cost of capital, WACC, in the Modigliani – Miller theory (Modigliani *et al.* 1958; 1963; 1966) is not minimal and valuation of the company capitalization is not maximal, as all financiers supposed up to this discovery: at some age of the company its WACC value turns out to be lower, than in Modigliani – Miller theory and company capitalization V turns out to be greater, than V in Modigliani – Miller theory. It was shown that, from the point of view of cost of attracting capital there are two types of dependences of weighted average cost of capital, WACC, on the company age n: monotonic descending with n and descending with passage through minimum, followed by a limited growth. In practice there are companies with both types of dependences of WACC on the company age n.

In this paper we have continued the study of the effect of the "golden age" of company and have investigated which companies have the "golden age", i.e. obey the latter type of dependence of WACC on n. With this aim we study the dependence of WACC on the age of company n at various leverage levels within wide spectrum of capital costs values as well as the dependence of WACC on leverage level L at fixed company age n. All calculations have been done within modern theory of capital cost and capital structure BFO by Brusov–Filatova–Orekhova (Brusov *et al.* 2011a,b,c,d,e; 2012 a,b; 2013 a,b,c; 2014 a,b; Filatova *et al.* 2008).

We have shown that existence of the "golden age" of company does not depend on the value of capital costs of the company, but depends on the difference value between equity, k_0, and debt, k_d, costs. The "golden age" of company exists at small enough difference between k_0 and k_d costs, while at high value of this difference the "golden age" of company is absent: curve WACC(n) monotonic descends with company age n. For the companies with the "golden age" curve WACC(L) for perpetuity limit lies between curves WACC(L) at n=1 and n=3, while for the companies without the "golden age" curve WACC(L) for perpetuity limit ($n=\infty$) is the lowest one. By other words, the ordering of curves WACC(L) is different for the companies with the "golden age" and without it.

In previous paper we have found also a third type of WACC(n) dependence: descending with passage through minimum, which lies below the perpetuity limit value, then going through maximum followed by a limited descending. We called this effect "Kulik effect" (this is last name of student, who have discovered this effect). In this paper we have found a variety of "Kulik effect": descending with passage through minimum of WACC, which lies above the perpetuity limit value, then going through maximum followed by a limited descending. We call this company age n, at which WACC has a minimum, which lies above the perpetuity limit value, "the silver age" of the company. It takes place at intermediate difference value between equity k_0 and debt k_d costs.

Because the cost of attracting capital is used in rating methodologies as discounting rate under discounting of cash flows, study of WACC behavior is very important for rating procedures. The account of effects of the "golden (silver) age" could change the valuation of creditworthiness of issuers.

Remind that, since the "golden age" of company depends on the company's capital costs, by controlling them (for example, by modifying the value of dividend payments, that reflect the equity cost), company may extend the "golden (silver) age" of the company, when the cost to attract capital becomes a minimal (less (above) than perpetuity limit), and capitalization of companies becomes maximal (above (below) than perpetuity assessment) up to a specified time interval.

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REFERENCES

Brusov P, Filatova T, Orehova N, and Kulik V (2015) The Golden Age of the Company: (Three Colors of Company's Time). Journal of Reviews on Global Economics, 4: 21-42. https://doi.org/10.6000/1929-7092.2015.04.03

Brusov P, Filatova T, Orehova N, Brusova A (2011a) Weighted average cost of capital in the theory of Modigliani–Miller, modified for a finite life–time company. Applied Financial Economics 21(11): 815-824.

Brusov P, Filatova P, Orekhova N (2013a) Absence of an Optimal Capital Structure in the Famous Tradeoff Theory! Journal of Reviews on Global Economics 2: 94-116.

Brusov P, Filatova P, Orekhova N (2014a) Mechanism of formation of the company optimal capital structure, different from suggested by trade off theory. Cogent Economics & Finance 2: 1-13. https://doi.org/10.1080/23322039.2014.946150

Brusov P, Filatova T, Orehova N *et al.* (2011b) From Modigliani–Miller to general theory of capital cost and capital structure of the company. Research Journal of Economics, Business and ICT 2: 16–21.

Brusov P, Filatova T, Eskindarov M, Orehova N (2012a) Influence of debt financing on the effectiveness of the finite duration investment project. Applied Financial Economics 22 (13) : 1043-1052.

Brusov P, Filatova T, Orehova N *et al.* (2011c) Influence of debt financing on the effectiveness of the investment project within the Modigliani–Miller theory. Research Journal of Economics, Business and ICT (UK) 2: 11-15.

Brusov P, Filatova T, Eskindarov M, Orehova N (2012b) Hidden global causes of the global financial crisis. Journal of Reviews on Global Economics 1: 106-111.

Brusov P, Filatova T, Orekhova N (2013b) Absence of an Optimal Capital Structure in the Famous Tradeoff Theory! Journal of Reviews on Global Economics 2: 94–116.

Brusov P.N., Filatova T. V. (2011d) From Modigliani–Miller to general theory of capital cost and capital structure of the company. Finance and credit 435: 2–8.

Brusov P Filatova T Orehova N Brusov P.P Brusova N. (2011e) From Modigliani–Miller to general theory of capital cost and capital structure of the company. Research Journal of Economics, Business and ICT 2: 16–21.

Brusov P Filatova T Orehova N (2014b) Inflation in Brusov–Filatova–Orekhova Theory and in its Perpetuity Limit – Modigliani – Miller Theory. Journal of Reviews on Global Economics 3: 175-185.

Brusov P Filatova T Orehova N (2013c) A Qualitatively New Effect in Corporative Finance: Abnormal Dependence of Cost of Equity of Company on Leverage. Journal of Reviews on Global Economics 2: 183-193.

Filatova T Orehova N Brusova A (2008) Weighted average cost of capital in the theory of Modigliani–Miller, modified for a finite life–time company. Bulletin of the FU 48: 68–77.

Brusova A (2011) A Comparison of the three methods of estimation of weighted average cost of capital and equity cost of company. Financial analysis: problems and solutions 34 (76): 36-42.

Modigliani F, Miller M (1958) The Cost of Capital, Corporate Finance, and the Theory of Investment. American Economic Review 48: 261–297.

Modigliani F, Miller M (1963) Corporate Income Taxes and the Cost of Capital: A Correction. American Economic Review 53: 147–175.

Modigliani F, Miller M (1966) Some estimates of the Cost of Capital to the Electric Utility Industry 1954–1957. American Economic Review 56: 333-391.

Myers S (1984) The Capital Structure Pussle. Journal of Finance39(3): 574-592. https://doi.org/10.1111/j.1540-6261.1984.tb03646.x

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