Inflation and Consumer Basket

Vladimir A. Popov*

Finacial University, Moscow, Russia

Abstract: Formation of consumer basket is investigated by means of probability theory. Inflation on the consumer market can be managed by reducing its rate and lowering inflation-related risks. The approach is based on the treating the inflation risks of particular ingredients of the consumer goods basket as component of the whole complex rather than separate units. The proposed management strategy is focused on the degree of correlation between the rates of price increase of the items in the basket. Portfolio theories of Markowitz and Tobin are used.

Keywords: Consumer market, inflation risk, correlation of price increase, expectation, covariance matrix.

I. INTRODUCTION

It is difficult to govern consumer basket under inflation. And it is especially difficult to make it than future prices are unknown. The risk of unpredictable price rises can significantly distract our plan of necessary purchases. Inflationary processes should necessarily be taken into account in business, state and household expenses. But this account as a rule use only predictions of inflation level and do not consider a volatile character interconnections of prices for various items in consumer basket.

There is similarity between compiling of consumer basket portfolio of securities compiling. Investor wants to maximize expected profit and minimize risk while buying securities. Ordinary customer wants not only to satisfy his necessities but to minimize expected prices and to minimize inflationary risk of jumping prices beyond some level.

Similar to portfolio of securities it is possible to govern consumer basket in order to lower expected cost and risk. In other words it is possible to reduce inflation effect an inflation risks by compiling consumer basket in proper way.

We use Markowitz and Tobin portfolio theories and consider inflationary risks of consumer basket ingredients in unity. Prices of various goods are interconnected. So there is correlation between rates of price rise of various goods. Prices on consumer market rise? But change in price of one commodity lead to change of in price of other commodities. So the risk of unexpected price jump of a whole consumer basket may be not so large even if risks of its isolated items price jump are large enough. One has to define proportions of ingredients in proper way. The process of diversification of consumer is realized in mathematical model. It is possible to calculate exact proportions of ingredients using mathematical formulas.

II. RANDOM CHARACTER OF PRICES

Statistical analysis of securities market made by Cowles and Working showed random character of price changing. If P_k is price of asset at end the period numbered k then $r_k = \ln \frac{P_k}{P_k - 1}$ and $R_k = r_1 + r_2 + \cdots + r_k$ are independent random values (Cowles, 1939), (Working, 1934). It contradicts to the contemporary opinion that prices have some rhythms, cycles, trends. The idea of random pricing was confirmed and developed in works of Kendal, Roberts and Osborn in which there are explained some statistically observed effects such as for example clustering (Kendal, 1953), (Roberts, 1959), (Osborn, 1959). Samuelson formulated hypothesis of effective market. He showed that expected price $P_k + 1$ of an asset at the end of a period $k+1$ equals conditional expectation of previous prices $P_1, P_2, \dots P_k$; $E(P_k + 1 / P_1, P_2, \dots P_k)$ (Samuelson, 1965).

The famous Markowitz work (Markowitz, 1952) laid the foundation of the theory of securities. The theory dials with investment optimization problem under condition of uncertainty and risk. Analysis from the point of view of probability theory shows very important value of correlation between profitableness of assets. The most fruitful idea of Markowitz theory is an idea of diversification while complying securities portfolio. It explains securities market behavior and gives practical recommendation to compiling optimal portfolio. We will use securities portfolio theory to investigation of consumer market.

We consider price changes of different goods in $S_{in} - \frac{P_{in} - P_{in}}{P_{io}}$ in n periods as normally distributed

*Address correspondence to this author at the Financial University, Moscow, Russia; Tel: +79265335744; Fax: +74992772321;
E-mail: vlapopov@gmail.com

E-ISSN: 1929-7092/18

random values with expectation $E_{in} = E(S_{in})$, standard deviation $\sigma_{in} = \sqrt{D(S_{in})}$ and correlation matrix $P_{ijn} = \rho(s_{in},; s_{jn})$. Price change of consumer of consumer basket is calculated by formula $S_n = x_1 s_{1n} + x_2 s_{2n} + \cdots + x_k s_{kn}$, where x_i is the share of item number i of a consumer basket, $x_1 + x_2 + \cdots + x_k = 1$. Expected inflation and dispersion of value S_n are calculated by formulas

$$\begin{cases} E(S_n) = \sum_{i=1}^{k} x_i E_{in} \\ D(H_n) = \sum_{i,j}^{k} \rho_{ijn}\sigma_{in}\sigma_{jn} \end{cases} \tag{1}$$

Here σ_j, σ_j are standard deviations of values s_{in}, s_{jn}.

Let us consider mean inflation $h = \frac{1}{n} S_n$ of consumer basket and mean inflation $q_i = \frac{1}{n} s_{in}$ of item i in one period. Let $E(h) = \frac{1}{n} E(H_{CM}) = \bar{h} = \mu$ be expected mean inflation of whole consumer basket in one period, and σ be standard deviation of whole consumer basket in one period. Let $E(q_i) = E(s_{in}) = \mu_i$ be expected mean inflation of item number i. Let $\sigma(q_i) = \sigma_i$ be standard deviation inflation of item number i and $\rho(q_i, q_j) = \rho_{ij}$ be correlation coefficient of two random values q_i and q_j. Than formulas (1) can written as follows.

$$\begin{cases} x_1 + x_2 + \cdots + x_n = 1 \\ \mu = \mu_1 x_1 + \mu_2 x_2 + \cdots + \mu_n x_n \\ \sigma^2 = \sum_{i,j=1}^{k} \rho_{ij}\sigma_i\sigma_j x_i x_j \end{cases} \tag{2}$$

It is possible change level of inflation μ risk of higher inflation σ by changing price shares x_i. We can find confidence interval of values h and S_n with given probability.

Now we let us investigate the situation than price changes of every item in consumer basket is unregulated. So for every item level of inflation is unpredicted and $\sigma_i \neq 0$.

First of all let us consider the consumer basket which consists only of two items. In this case there is only one correlation coefficient $\rho = \rho_{12}$. Les us denote $\mu_1 = t$ than $\mu_2 = 1 - t$. System of equations (2) will be as follows.

$$\begin{cases} \mu = t\mu_1 + (1-t)\mu_2 \\ \sigma^2 = t2\sigma_1^2 + 2\rho\sigma_1\sigma_2 t(1-t) \\ t \in [0;1], \rho \in [-1;1] \end{cases} \tag{3}$$

System (3) can be considered as definition of a function $\mu = f(\sigma)$. If $\mu \neq \pm 1$ the curve is hyperbola with branches directed along σ. Coordinates of its vertex $(\mu_0; \sigma_{\min})$ can be calculated by standard mathematical methods [3]. The risk $\sigma_{\min}$ of corresponding consumer basket is minimal risk among risks of all possible consumer baskets. As in portfolio theory we call consumer basket with standard deviation $\sigma = \sigma_{\min}$ minimal consumer basket. Price shares of this basket can be calculated by standard mathematical methods too [3]. On Figure **1** horizontal axe is axe of risk and vertical axe is axe of level of inflation. Point M is point of minimal risk $\sigma_{\min}$.

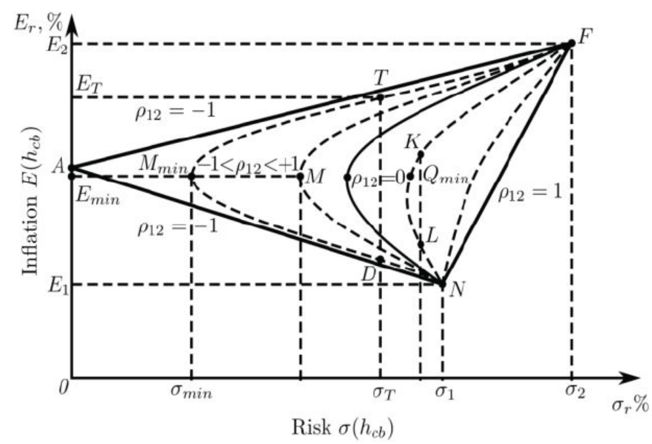

Figure 1: Values of risc and inflation with various structure of consumer basket correlation of ingredients.

One can solve system of equation (3) and calculate price shares x_1 and x_2.

$$\begin{cases} x_2 = t = \dfrac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2} \\ x_2 = 1 - t = \dfrac{\sigma_1^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2} \end{cases} \tag{4}$$

For given values $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$ we can change price shares x_1, x_2 and lessen expected level of inflation μ of the consumer basket and its risk σ. Let us consider the following model example.

Monthly forecasting of price rising for two items equal to $\mu_1 = 0.65\%$ and $\mu_2 = 0.99\%$ respectively. Risks of deviation from this expected level of inflation equal to $\sigma_1 = 3.23\%$ and $\sigma_2 = 4.86\%$ for first and second items respectively. Correlation forecast is equal to $\rho = 0.39$.

Initial plan of consumption is as follows. Price share for the first ingredient of consumer basket equals $x_1 = 0.7$ and price share for the second ingredient of consumer basket equals $x_2 = 0.3$. It easy to calculate by formulas (3) that $\mu = 0.75\%, \sigma = 3.12\%$.

Let us check point $K(\mu = 0.75; \sigma = 3.12)$ (See Figure **1**). Standard deviation of inflation level μ can be found by formulae $\sigma = \frac{\mu\% \cdot \sigma\%}{100}$. So $\sigma = \frac{0.75 \cdot 3.12}{100} = 0.02\%$. Annual inflation rate is equal to $\mu_y = 12\mu = 9\%$. Risk of annual rate deviation is equal to $\sigma_y = \sigma\sqrt{12} = 3.12 \cdot \sqrt{12} = 10.81$. So standard annual deviation is equal to $\sigma_y = \frac{9 \cdot 10.81}{100} = 0.97\%$. According to (3) $H_{CMy} \in (6.08\%; 11.92\%)$. σ rule" we have confidence interval with confidence probability 0.9973

Structure of consumer would be better (cheaper and less risky) if parameters of consumer basket correspond to point $Q_{\min}$ which lies on arc $NLQ_{\min}KF$ (this arc is determined by values of μ and σ). But to point $Q_{\min}$ corresponds to another structure of consumption. From formulas (4) it follows that $x_{1\min} = 0.8; x_{2\min} = 0; h = \mu_{\min} = 0.72; \sigma = \sigma_{\min} = 3.1\%$. Now we can forecast annual inflation $\mu_y = 12\mu = 12 \cdot 0.72 = 8.64\%;$ $\sigma = 0.02;$ $\sigma_y = \frac{8.64 \cdot 10.81}{100} = 0.93\%$; $h \in (0.68; 0.78);$ $H_{CMy} \in (5.86\%; 11.42\%).$

It would even better to choose point L where with the same risk as in point K inflation is less than in point $Q_{\min}$. But proportions of ingredients corresponding to that point are such that consumption of second ingredient almost disappears, and that is no good for buyer. If we change ingredients proportions in order to go along the way $FTM_{\min}DN$ (this is more visually demonstrative in comparison with the way $FKQ_{\min}lM$) from point $M_{\min}$ to D than we will reduce inflation but increase risk. One has to calculate inflations and risks for various structures of consumer basket in order to choose optimal proportion of ingredients.

III. PRICE CHANGES OF SOME ITEMS IN CONSUMER BASKET IS PREDERTENENED

Above we discussed situation where the theory of Markowitz portfolio can be applied. I. e. we considered positively determined correlation matrixes (ρ_{ij}). This supposition often occurs in securities portfolio and can be applied for certain types of consumer basket. Generally correlation matrix is degenerated. For many goods inflation is absent or strictly regulated i. e. have zero deviation (σ = 0). Let us unite this goods and services as one ingredient and define it as component of consumer basket without risk. In portfolio theory a model with component without risk is well studied and called Tobin portfolio. (Tobin, 1965).

Let us divide consumer basket in two parts. First part with share x_0 has no risk and its expected inflation equals μ_0. Second part which is called "marketing part" consists of n components with shares $x_1, x_2, \ldots, x_n$ inflationary expectations $\mu_1, \mu_2, \ldots, \mu_n$ risks $\sigma_1, \sigma_2, \ldots, \sigma_n$ and correlation matrix (ρ_{ij}). Mean inflation and risk of a consumer basket as a whole are calculated by following formulas.

$$\mu = \mu_0 x_0 + \mu_1 x_1 + \mu_2 x_2 + \cdots \mu_n x_n,$$
$$\sigma = \sum_{i,j=1}^{n} \rho_{ij}\sigma_i\sigma_j x_i x_j,$$
$$x_0 + x_1 + x_2 + \cdots x_n = 1.$$

We want to maximize profit μ while investigating securities portfolio. But we are minimizing inflation μ while investigating consumer basket. We use the equation of minimal boundary from Tobin's portfolio theory [9]. $\sigma^2 = \left(\frac{\mu - \mu_0}{d}\right)^2$, where $d = \sqrt{\alpha\mu_0^2 - 2\beta\mu_0 + \gamma}$, $\alpha = I^T V^{-1} I$, $\beta = \vec{I}^T V^{-1}\vec{\mu}, \gamma = \vec{\mu}^T V^{-1}\vec{\mu}, V = (\rho_{ij})$ is correlation matrix of risky ingredients, $\vec{I} = (1; 1; \ldots; 1)^T$, $\vec{\mu} = (\mu_1; \mu_2; \ldots; \mu_n)^T$.

The securities portfolio profit of risky security is greater than profit security without risk, $\mu_i \geq \mu_0$. For consumer basket it is often vice versa and what's more we can guess that $\mu_i \leq \mu_0$. So we have an equation of minimal boundary $\sigma = \frac{\mu_0 - \mu}{d}$.

The risky part of securities basket $X = (x_1; x_2; \ldots; x_n)^T$ is calculated by formulae $\vec{X} = \frac{\mu_0 - \mu}{d^2} V^{-1}(\vec{\mu} - \mu_0\vec{I})$ and share of ingredient without risk equals $x_0 = 1 - \vec{I}^T\vec{X}$.

IV. CONCLUSIONS

A Markowitz and Tobin theory allows choosing the consumer basket of minimal inflation risk. But such basket has purely theoretical application as it demands the serious change customer's habitual consumption structure. So given above formulas are aimed to show the possibility of sufficient changing of consumer's consumption structure by substituting items of arbitrary chosen basket for the higher quality analog by reducing expected price and risk of price jumps.

But the serious change in consumer's consumption structure can be possible only by state regulation. Such conclusion does not contradict to classical Keynes's evaluations of state regulations in economy. State should implement tax, expenditures, and monetary regulations to eliminate any economic instability. So as for Keynes state should act in the situations where private business fails. And the regulations of inflation risks and price increases are one of the state priorities too.

In some way this work is related to works (Popov, Semenov, 2009), (Popov, Semenov, 2018), (Popov, 2018), (Chistiakova, Sukhorukova, 2018).

We hope that idea of using the theory of securities portfolio to investigate consumer basket formation is fruitful and deserves further development. It is possible to take into consideration utility function in order to compile consumer basket precisely. On the other hand changing of quantity of goods in consumer market should be restricted. But choosing of utility function and restrictions on amount of items should be made differently for different consumer baskets. And analysis of special consumer baskets is the subject of other investigations.

REFERENCES

Cowles A, (1939) "Can stock market focasters forcast?", Econometrica.V.I.P 309-324.

Kendal MG, (1953) "The analisis of Economic Time Series", Journalof the Royal Statistical Society 96: 11-25. https://doi.org/10.2307/2980947

Markowitz H, (1952) "Portfolio selection", in Journal of Finance 7: 77-91.

Osborn ME, (1959) "Brownian motion in stock market", Operations reseaarch 7: 145-173. https://doi.org/10.1287/opre.7.2.145

Popov VA, (2018) "Inflation and Cash", Journal of Reviews on Global Economics 7: 152 – 156 https://doi.org/10.6000/1929-7092.2018.07.13

Popov VA, Semenov VP, (2009) "The method of Calculation of Inflation," Financial Business, April.

Popov VA, Semenov VP, (2010) "Inflation risk modelling", Science results. South of Russia. Serie "Mathematical forum". Vladikavkaz. 4: 353-360

Roberts HV, (1959) "Stock-market "Patterns" and Financional Analisis. Metodological Suggestions", Journal of Finance 14: 1-10.

Samuelson FA, (1965) "Rational theory of warrant pricing", industrial Management Review 6: 1-10.

Sukhorukova IV, Chistiakova NA (2018) "Optimization of the Formation of the Capital Structure of the Insurance Company, Taking into Account the National Specifics of Insurance", Journal of Reviews on Global Economics 7: 146 – 151. https://doi.org/10.6000/1929-7092.2018.07.12

Tobin J, (1965) "The theory of portfolio selection. The the thory of interest rates", Ad. By F.H. Hhan and F.P.A. Brechling, London.

Working H, (1934) "A random −difference series for use in the analisis of time series", Journal of American Statistical association 29: 11-24. https://doi.org/10.1080/01621459.1934.10502683

Received on 30-05-2018 Accepted on 26-09-2018 Published on 12-11-2018

DOI: https://doi.org/10.6000/1929-7092.2018.07.40