BFO Theory with Variable Profit in Case of Advance Payments of Tax on Profit

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Abstract: The Brusov–Filatova–Orekhova (BFO) theory is generalized for the simultaneous account of variable company profit and advance tax on income payments. The generalized BFO formula for the WACC, has been derived. The dependence of WACC, discount rate, WACC–g (here g is growth rate), company capitalization, V, the equity cost, ke, on leverage L at various values of g, on debt cost, kd, and on age of the company, n, is studied. It is shown, that WACC, is no longer a discount rate. This role passes to WACC–g, which decreases with g, while the company's value increases with g. The tilt of curve k(L) growths with g. It is found that at the growth rate g < g* the tilt of the curve ke(L) is negative. This changes significantly the company's dividend policy principles. WACC(L) as well as the discount rate, WACC–g, decrease with the increase of debt cost kd. V (L) at all values of k_d increases with leverage L, as well V(L) increases with k_d. This means that tax shield advantages the decrease of the cost of raising capital. Examining the main financial parameters of the company at the positive (g=0.2) and negative (g=–0.2) growth rates, we found a huge difference in their behavior. This allows you to explore companies with growing profits and companies with decreasing profits, as well investigate the financial state of the companies whose profits rise and fall in different periods.

Keywords: Generalized BFO theory, variable income, advance tax on income payments, company value, WACC, cost of equity.

1. INTRODUCTION

Two main capital structure theories – Brusov–Filatova–Orekhova (BFO) and its perpetuity limit – theory by Nobel Prize winners Modigliani and Miller – consider the companies with constant profit, while in practice profit of the company is, of cause, variable. Recently the Modigliani – Miller (MM) theory has been generalized for the case of variable profit (Brusov *et al*. 2021), and in current paper we have generalized for the first time BFO theory for the companies with variable profit and advance tax on profit payments. After such a generalization, the applicability of the BFO theory, which is valid for companies of any age, significantly expands in practice, in particular, in corporate finance, in investments, in business valuation, in banking, in ratings, etc. (Brusov *et al*. 2023).

1.1. Review of Literature

The capital structure is the ratio between the equity and debt value. Whether the capital structure affect main financial parameters and if so, how?

The main tasks solved by the company's management is to determine the optimal capital structure, in which the cost of raising capital is minimal, and the value of the company V is maximum.

The first quantitative work on this topic was that of (Modigliani and Miller 1958). Before this, there was the so-called traditional (empirical) approach.

1.2. The Basis of the Traditional Approach (TA)

In TA, WACC and company value V depend on the level of leverage L (capital structure). Debt is cheaper than equity, because the former has less risk, since in the event of bankruptcy, the claims of creditors are satisfied before the shareholders' ones.

Thus, increasing the share of cheaper debt capital in the total capital structure to a limit that does not violate financial stability and does not increase the risk of bankruptcy reduces WACC and increases the value of the company, V.

A further increase in debt financing leads to a violation of financial stability and to the risk of bankruptcy increase. WACC goes up, while company capitalization, V, goes down. The optimal capital structure is formed as a result of competition between the advantages of debt financing with a low level of leverage and its disadvantages with a high level of leverage. The trade-off theory has come to similar conclusions.

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1.3. Modigliani–Miller Theory

1.3.1. Modigliani–Miller Theory Without Taxes

(Modigliani and Miller 1958), under numerous restrictions, came to the conclusion that, without taxes, the value of a company is constant and equal to

$$V = V_0 = \frac{EBIT}{k_0}\,, \tag{1}$$

where EBIT is earnings before interest and taxes, k_0 is the discount rate. This contradicts the findings of TA.

From (1) one obtains for WACC:

$$WACC = k_0\,. \tag{2}$$

k0 is the equity cost at L=0.

Using

$$\mathrm{WACC} = k_0 = k_\mathrm{e} w_\mathrm{e} + k_\mathrm{d} w_\mathrm{d}. \tag{3}$$

and (1) we get for equity cost, ke

$$k_\mathrm{e} = \frac{k_0}{w_\mathrm{e}} - k_\mathrm{d}\frac{w_\mathrm{d}}{w_\mathrm{e}} = \frac{k_0(S+D)}{S} - k_\mathrm{d}\frac{D}{S} = k_0 + (k_0 - k_\mathrm{d})\frac{D}{S} = k_0 + (k_0 - k_\mathrm{d})L \tag{4}$$

were, L stands for leverage level, D is debt capital value of the company; WACC stands for weighted average cost of capital; S stands for equity value; k_d and w_d stands for the debt cost and share; k_e and w_e stands for the equity capital cost and share. From (4) it follows that the equity cost increases linearly with the level of leverage.

1.3.2. Modigliani–Miller Theory With Taxes

Taking into account the income tax,

In 1963 Modigliani and Miller (Modigliani and Miller 1963; Modigliani and Miller 1966), accounting the tax on profit, have obtained for the levered company value, V,

$$V = V_0 + D\cdot t \tag{5}$$

where D stands for debt value, V_0 stands for the unlevered company value, and T stands for the tax on income rate.

From (5) one gets

$$WACC = k_0\cdot(1 - w_d t) \tag{6}$$

One gets from (6) for equity cost ke

$$k_e = k_0 + L\cdot(k_0 - k_d)(1-t) \tag{7}$$

Formula (7) (MM with taxes) differs from formula (4) (MM without taxes) by the factor (1–t) (tax corrector). It is less than unity, so the the ke(L) curve slope decreases when taxes are included.

1.4. Unification of Capital Asset Pricing Model (CAPM) with Modigliani–Miller Model

The Modigliani–Miller theory, with accounting taxes has been united with CAPM (Capital Asset Pricing Model) in 1961 by Hamada (Hamada R., 1969). For the cost of equity of leveraged company the below formula has been derived

$$k_\mathrm{e} = k_\mathrm{F} + (k_\mathrm{M} - k_\mathrm{F})b_\mathrm{U} + (k_\mathrm{M} - k_\mathrm{F})b_\mathrm{U}\frac{D}{S}(1-T), \tag{8}$$

Here b_U is the β–coefficient of the unlevered company. First term represents risk–free profitability k_F, second term - business risk premium, $(k_\mathrm{M} - k_\mathrm{F})b_\mathrm{U}$, and third term - financial risk premium $(k_\mathrm{M} - k_\mathrm{F})b_\mathrm{U}\frac{D}{S}(1-T)$.

In the case of a unlevered company (D = 0), the financial risk (the third term) is zero, and its shareholders receive only a business risk premium.

1.5. Miller Model

Corporate and individual taxes were taken into account by Miller 1977, and the following formula was obtained for the value of a company without borrowed funds, V_U,

$$V_\mathrm{U} = \frac{\mathrm{EBIT}(1-T_\mathrm{C})(1-T_\mathrm{S})}{k_0}. \tag{9}$$

Here T_C stands for the corporate tax on income rate, T_S stands for the tax rate on profits of an individual investor from his ownership by stock of corporation, T_D–tax rate on interest profits from the provision of investor–individuals of credits to other investors and companies. A factor $(1-T_\mathrm{S})$ accounts the individual taxes.

1.6. Brusov–Filatova–Orekhova (BFO) theory

One of the most important limitations of the Modigliani-Miller theory, removed in 2008 by Brusov-Filatova-Orekhova (Brusov *et al*. 2018; Brusov *et al*. 2020), is the perpetual nature of the company. The authors generalized the Modigliani-Miller theory for companies of arbitrary age and obtained the famous BFO formula for WACC.

$$\frac{1-(1+WACC)^{-n}}{WACC}=\frac{1-(1+k_0)^{-n}}{k_0\left[1-w_dT\left(1-(1+k_d)^{-n}\right)\right]} \quad (10)$$

To get Modigliani–Miller limit one should substitute $n \to \infty$.

A number of new effects, obtained in Brusov–Filatova–Orekhova theory (Brusov *et al*. 2018; Brusov *et al*. 2020), are absent in Modigliani–Miller theory (Brusov *et al*. 2021; Modigliani and Miller 1958, 1963).

The BFO theory destroyed some of the main principles of financial management that have existed for many decades: among them, one of the cornerstones of the formation of an optimal capital structure - trade-off theory -, the failure of which was proved by Brusov *et al.* 2018, 2020.

1.7. Alternative Expression for WACC

Alternative formula for the WACC, different from Modigliani – Miller one has been derived in (Farber *et al.* 2006; Fernandez P, 2006; Berk and DeMarzo 2007; Harris and Pringle 1985) from the WACC definition and the balance identity (see Farber *et al.* 2006):

$$WACC = k_0(1-w_dT)-k_dtw_d+k_{TS}tw_d\,, \quad (11)$$

where k_0, k_d and k_{TS} are the expected returns respectively on the unlevered company, the debt and the tax shield.

Some additional conditions are required equation (11) practical applicability. If the WACC is constant over time, as it stated in (Farber *et al.* 2006) the levered company capitalization is found by discounting with the WACC of the unlevered company.

In textbooks (Berk and DeMarzo 2007; Harris and Pringle 1985) formulas for the special cases, where the WACC is constant, could be found.

In 1963 Modigliani and Miller assume that the debt value D is constant. Then, as the expected after–tax cash–flow of the unlevered firm is fixed, V_0 is constant as well. By assumption, k_{TS} = k_D and the value of the tax shield is TS = tD. Thus, the capitalization of the company V is a constant and the alternative formula (11) becomes a formula for a constant WACC:

$$WACC = k_0(1-w_dT)$$

Because the debt kd and the tax shield k_{TS} have debt nature it seems reasonable that the expected returns on they are equals as suggested by "classical" Modigliani – Miller (MM) theory, which has been modified by Brusov *et al*. 2021 for cases of practical meanings.

1.8. Trade–Off Theory

The world famous trade–off theory was the cornerstone for many decades and is popular now (see, for example (Frank and Goyal 2009; Sheridan and Wessels 1988)).

However, in 2013 the trade–off theory' bankruptcy has been proved by Brusov *et al*. 2018, 2020. The risky debt financing does not lead to the WACC growth, and WACC still decreases with leverage. Thus the minimum in the WACC(L) curve and of maximum in the V(L) are absent. This means, that within the world–famous trade–off theory the optimal capital structure is absent. Brusov *et al*. 2018, 2020 gave an explanation for this fact.

Modigliani-Miller stressed the importance of the tax shield. In (Sheridan and Wessels 1988; Harry and Masulis 1980; Bradley *et al.* 1984; Graham JR, 2003), the study of the tax shield was continued, including non–debt tax shield (Graham JR, 2003). It was shown, that the effects of substitution between debt and non–debt tax shields are suppressed by a positive relation of this type.

The Brusov–Filatova–Orekhova (BFO) theory, its methodology, and results are widely known (see, for example, (Dimitropoulos P, 2014; Luiz and Cruz 2015; Barbi M, 2011; Franc–Dąbrowska *et al.* 2021; Angotti *et al*. 2018; Vergara–Novoa *et al*. 2018; Mundi *et al*. 2022; Sadiq *et al*. 2022; Becker DM, 2022; El–Chaarani *et al*. 2022)). A lot of authors (Angotti *et al*. 2018; Vergara–Novoa *et al*. 2018; Mundi *et al*. 2022) use the BFO theory in practice.

A few years ago the adaptation of the two main theories of the capital structure (Brusov–Filatova–Orekhova and Modigliani–Miller) to the established financial practice of the company's functioning by taking into account the real conditions of their work has been done by the authors of this paper (Brusov and Filatova 2021; Brusov *et al*. 2022; Brusov and Filatova 2022; Brusov and Filatova 2023).

2. MODIFICATION OF THE BRUSOV–FILATOVA–OREKHOVA (BFO) THEORY TO THE CASE OF COMPANIES WITH VARIABLE INCOMES AND ADVANCE PAYMENTS OF TAX ON PROFIT

2.1. The Financially Dependent Company Value, V

Below, for the first time, we generalize the BFO theory for the case of variable profit. Assuming that income for the period grows with rate g, we derive the formula for a financially dependent company value.

$$V=\frac{CF}{1+WACC}+\frac{CF(1+g)}{(1+WACC)^2}+\frac{CF(1+g)^2}{(1+WACC)^3}+\ldots+\frac{CF(1+g)^{n-1}}{(1+WACC)^n} \quad (12)$$

where *CF* is an annual profit of company.

(12) is geometric progression with denominator

$$g=\frac{(1+g)}{(1+WACC)}. \quad (13)$$

Summing (12), one gets for V

$$V=\frac{CF}{1+WACC}\cdot\frac{1-\left(\frac{1+g}{1+WACC}\right)^n}{1-\frac{1+g}{1+WACC}}=\frac{CF}{WACC-g}\cdot\left(1-\left(\frac{1+g}{1+WACC}\right)^n\right) \quad (14)$$

In perpetuity (MM) limit ($n\to\infty$) one gets for V,

$$V=\frac{CF}{WACC-g} \quad (15)$$

This means that WACC–g plays the role of the discount rate, not WACC.

2.2. The Value of a Financially Independent Company, V_0

Let us, assuming that profit for the period grows with the rate g, derive formula for a financially independent company value V_0,

$$V_0=\frac{CF}{1+k_0}+\frac{CF(1+g)}{(1+k_0)^2}+\frac{CF(1+g)^2}{(1+k_0)^3}+\ldots+\frac{CF(1+g)^{n-1}}{(1+k_0)^n} \quad (16)$$

(16) is geometric progression with denominator

$$g=\frac{(1+g)}{(1+k_0)} \quad (17)$$

Summing (16), one gets for a financially independent company value, V

$$V_0=\frac{CF}{1+k_0}\cdot\frac{1-\left(\frac{1+g}{1+k_0}\right)^n}{1-\frac{1+g}{1+k_0}}=\frac{CF}{k_0-g}\cdot\left(1-\left(\frac{1+g}{1+k_0}\right)^n\right). \quad (18)$$

In perpetuity (MM) limit ($n\to\infty$) one gets for a financially independent company value, V_0,

$$V_0=\frac{CF}{k_0-g}. \quad (19)$$

This formula shows, that discount rate is k_0–g, and not k_0.

2.3. The Tax Shield Value

The tax shield for n–year company for advance payments of tax on income is equal

$$(TS)_n=tk_dD+\frac{tk_dD}{1+k_d}+\ldots+\frac{tk_dD}{(1+k_d)^{n-1}} \quad (20)$$

(20) is geometric progression with denominator

$$g=\frac{1}{(1+k_d)} \quad (21)$$

Summing (20), one gets for tax shield

$$(TS)_n=tk_dD\cdot\frac{1-(1+k_d)^{-n}}{1-\frac{1}{1+k_d}}=Dt\left(1-(1+k_d)^{-n}\right)\cdot(1+k_d) \quad (22)$$

$$(TS)_n=Dt\left(1-(1+k_d)^{-n}\right)\cdot(1+k_d) \quad (23)$$

Generalizing the first Modigliani – Miller theorem for finite company age, we get

$$V=V_0+(TS)_n \quad (24)$$

Substituting

$$D=w_dV, \quad (25)$$

we arrive to the following expression

$$V\left(1-w_dt\left(1-(1+k_d)^{-n}\right)\cdot(1+k_d)\right)=V_0 \quad (26)$$

Substituting here the values of a financially independent company, V_0 (18) and of a financially dependent company, *V*, (14) one gets

$$\frac{CF\cdot\left(1-\left(\frac{1+g}{1+WACC}\right)^n\right)\cdot\left(1-w_dt\left[1-(1+k_d)^{-n}\right]\cdot(1+k_d)\right)}{WACC-g}=\frac{CF\cdot\left(1-\left(\frac{1+g}{1+k_0}\right)^n\right)}{(k_0-g)} \quad (27)$$

From (27) we arrive to the BFO equation for the case of the company variable profit

$$\frac{1-\left(\frac{1+g}{1+WACC}\right)^n}{WACC-g} = \frac{1-\left(\frac{1+g}{1+k_0}\right)^n}{(k_0-g)\cdot\left(1-w_d t\left[1-(1+k_d)^{-n}\right]\cdot(1+k_d)\right)}. \quad (28)$$

This formula is the main theoretical result of the article.

For the Modigliani–Miller theory with the variable profit (Brusov *et al*. 2021) one gets

$$WACC-g=(k_0-g)\cdot(1-w_d t\cdot(1+k_d)) \quad (29)$$

$$WACC=(k_0-g)\cdot(1-w_d t\cdot(1+k_d))+g \quad (30)$$

3. RESULTS AND DISCUSSIONS

Below we study within Microsoft Excel the impact of a growth rate g on the company financial parameters (WACC; WACC-g; V; ke) using the formula (28), under investigating their dependences on the level of leverage L. We present the results for the following financial parameters of the company:

k_0=0.26; k_d=0.22; t=0.2; *CF*=100; n=5

Here k_0 stands for the equity cost at zero leverage level; k_d stands for the debt cost; t stands for the tax on profit; *CF* stands for profit per period; n stands for the company age; L stands for the leverage level.

Note that the qualitative effect of the growth rate g on the main financial parameters is similar, while the results for different parameters are different numerically.

3.1. Five–Year Company

3.1.1. Weighted Average Cost of Capital, WACC

As it could be seen from Figure **1** all curves WACC(L) for different values of g start from one point (0; k_0=0.26). All curves WACC(L) decrease with leverage level L at all g values. The curves WACC(L) increase with g.

The five–year company results differ from the results for a perpetual limit – the theory of Modigliani

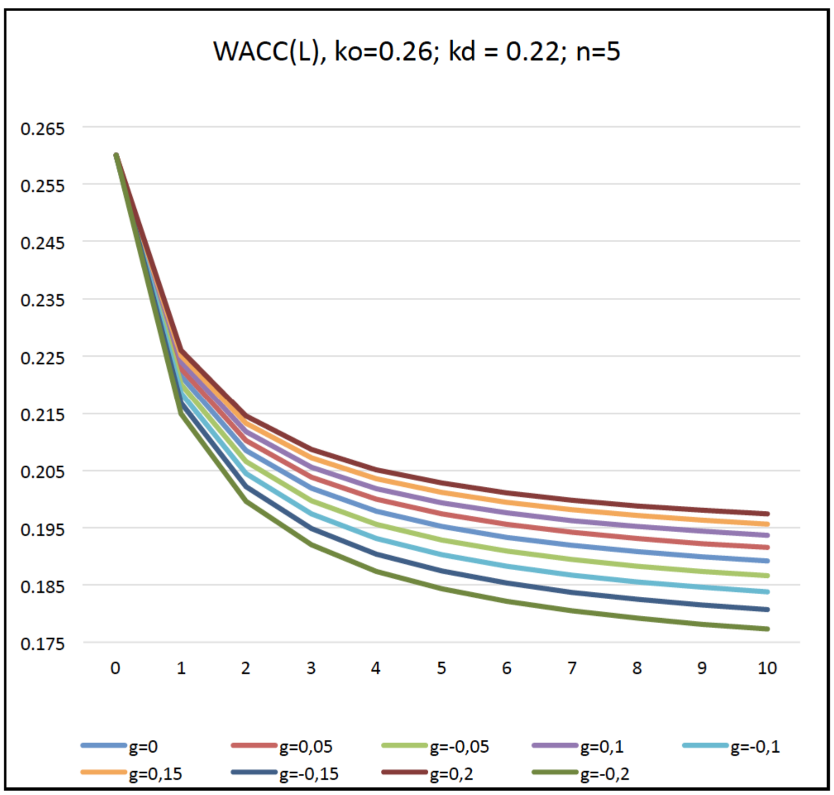

Figure 1: The dependence of WACC on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2; 0.15;0.1; 0.05; 0.0; –0.05;–0.1; –0.15; –0.2 for five–year company.

and Miller (Brusov *et al*. 2021). In the latter case, the WACC(L) curves decrease with the level of leverage L at $g<k_0$ and increase at $g>k_0$. k_0 is the threshold value g separating the increasing WACC(L) curves from the decreasing ones, and for $g=k_0$ WACC=const=k_0. In the first case (BFO theory), the WACC(L) curves decrease with increasing leverage L for all values of the growth rate g. The WACC(L) curves increase with the rate g both in the Brusov–Filatova–Orekhova theory and in the Modigliani and Miller theory. This means that WACC is no longer a discount rate. As it will be seen below, the role of the discount rate play WACC–g and WACC–k_0.

Below we study the dependence of WACC–g on L in the Generalized theory of Brusov–Filatova–Orekhova (the GBFO theory) at k_0=0.18 and different values of g (–0.2; –0.1; 0.0 ; 0.1; 0.2) for two– and four – year company.

3.1.2. Calculations of the Discount Rate, WACC–g

As it is seen from Figure **2** the curves (WACC–g)(L) decrease with leverage level L at all g values. The curves (WACC–g)(L) decrease with growth rate, g.

The explanation of the behavior of the (WACC–g)(L) curves is as follows: all WACC(L) curves originate from the same point (L=0; WACC=0.26). The (WACC–g)(L) curves will be ordered as follows for L=0: the larger g, the lower the starting point and hence the entire graph lies, since the curves do not intersect. As we'll see below the decrease of (WACC–g)(L) with growth rate, g, will lead to increase of the company value, V, with g.

3.1.3. Calculations of the Company Value, V

It is seen from Figure **3**, that the company value V at fixed growth rate g increases with leverage level L. The company value V increases with growth rate g as well.

Below we study the equity cost, k_e, dependence on leverage level L and on growth rate, g, at k_0 =0.18; k_d=0.16 and g = 0;$\pm$0.1; $\pm$0.2.

3.1.4. Calculations of the Equity Cost, ke

As it is seen from Figure **4** the equity cost, ke, practically linearly grows with leverage level L at all growth rate g values. The tilt angle ke(L) increases with g.

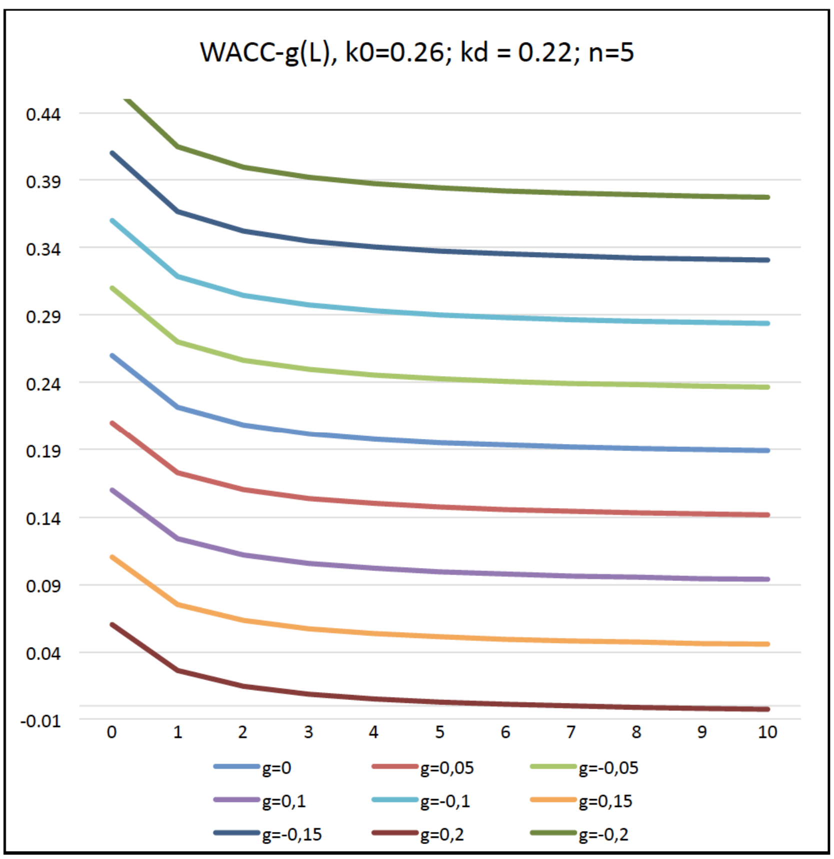

Figure 2: The dependence of discount rate, WACC–g, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2; 0.15;0.1; 0.05; 0.0; –0.05;–0.1; –0.15; –0.2 for five–year company.

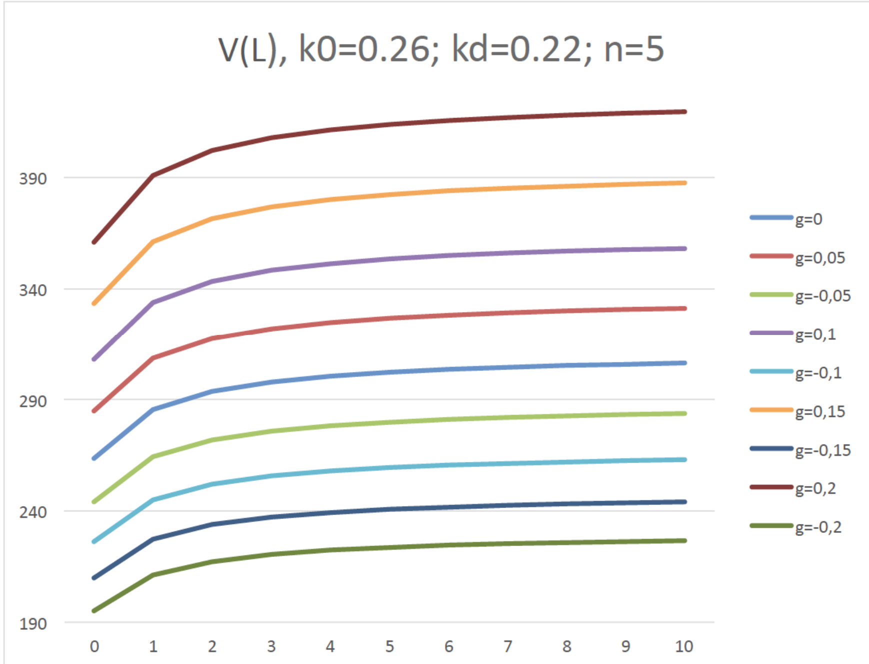

Figure 3: The dependence of discount rate, WACC–g, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2; 0.15;0.1; 0.05; 0.0; –0.05;–0.1; –0.15; –0.2 for five–year company.

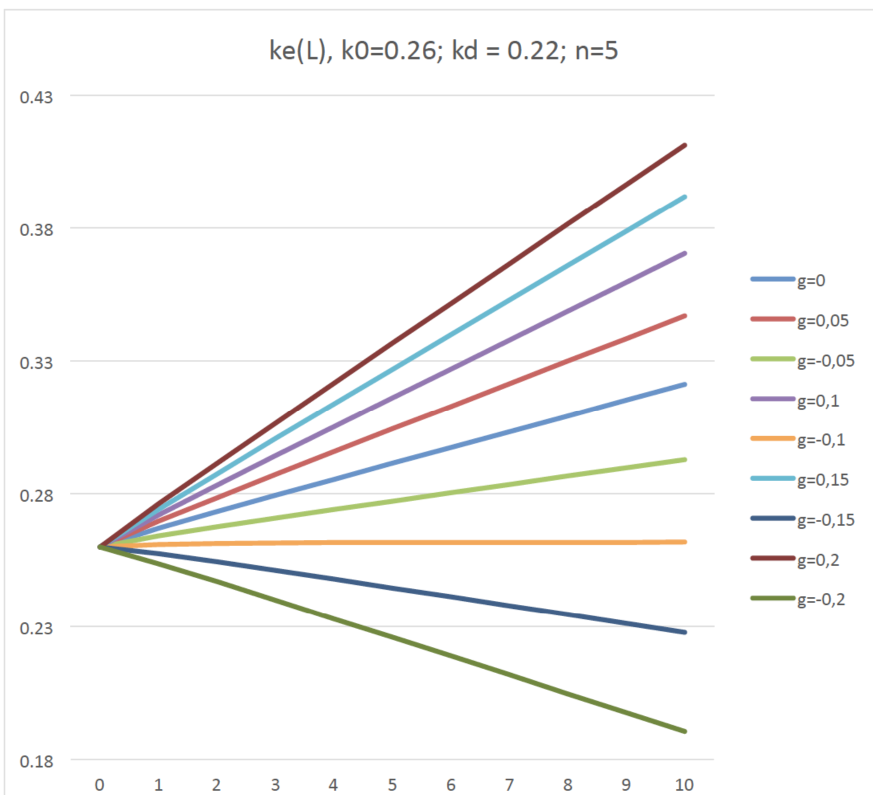

Figure 4: The dependence of equity cost, ke, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2; 0.15;0.1; 0.05; 0.0; –0.05;–0.1; –0.15; –0.2 for five–year company.

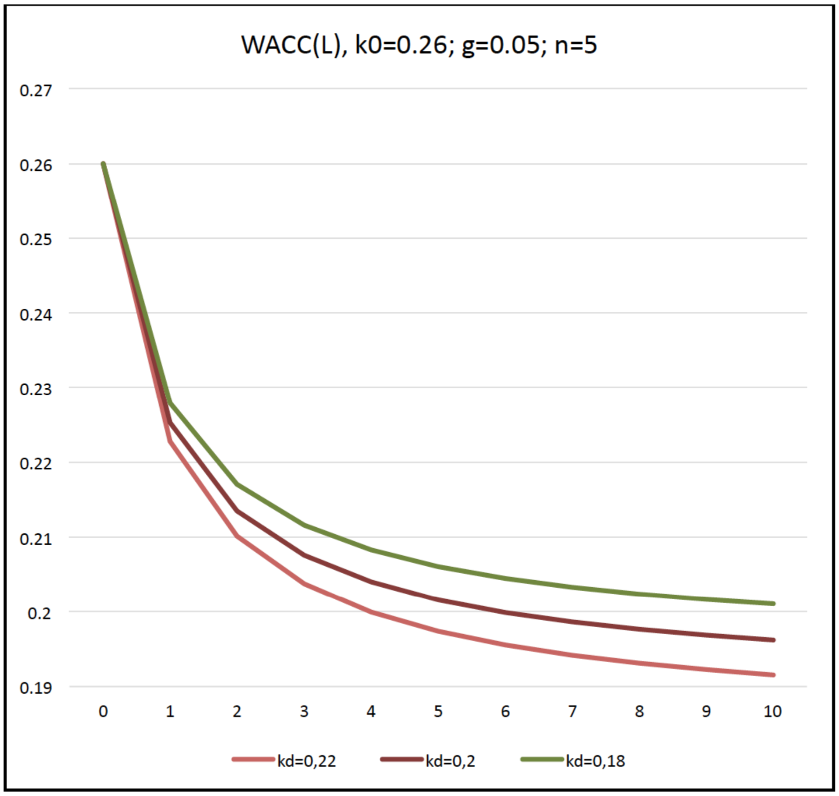

Figure 5: The dependence of WACC on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; 0.2; 0.18 t=0.2; g=0.05 for five–year company.

3.2. Study the Dependence of Financial Indicators on k_d

3.2.1. The Weighted Average Cost of Capital, WACC

It is seen from Figure **5** that all curves WACC(L) at all values of k_d start from one point (0; 0.26). WACC(L) decrease with leverage level L at all values of k_d. WACC(L) decrease with the increase of debt cost k_d. This means that tax shield advantages the decrease of the cost of raising capital.

3.2.2. The Discount Rate, WACC–g

It is seen from Figure **6** that all curves of discount rate (WACC–g) (L) at all values of k_d start from one point (0; 0.21). (WACC–g) (L) decrease with leverage level L at all values of k_d. (WACC–g)(L) decrease with the increase of debt cost k_d. This means that the tax shield tends to lower the value of the discount rate (WACC–g) and hence (as we will see below in 3.2.3) increase the value of the company, V.

3.2.3. The Company Value, V

From Figure **7** it follows that all curves of company value V (L) at all values of k_d start from one point (0; 285). V (L) increases with leverage level L at all values of k_d. V(L) increases with the increase of debt cost k_d. This means that tax shield advantages the increase of the company value, V.

It follows from Figure **8** that all curves of equity cost, ke(L), at all values of k_d start from one point (0; 0.26) and k_e increases with leverage level L at all values of k_d. The slope of the straight line ke(L) decreases with the cost of debt kd. This means that debt cost kd impact the dividend policy of the company, because the equity cost ke determines the economically justified amount of dividends.

3.3. Impact of Company Age, n, on Main Financial Indicators of the Company

Below we study the impact of company age, n, on main financial indicators of the company: WACC; WACC–g; V; ke. We investigate the dependence of WACC; WACC–g; company value, V, and equity cost, ke on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2 and g=–0.2 for five–year and ten–year companies. We found a huge difference between the behavior of the main financial indicators of the company with a positive and negative growth rate g. This allows you to explore companies with growing

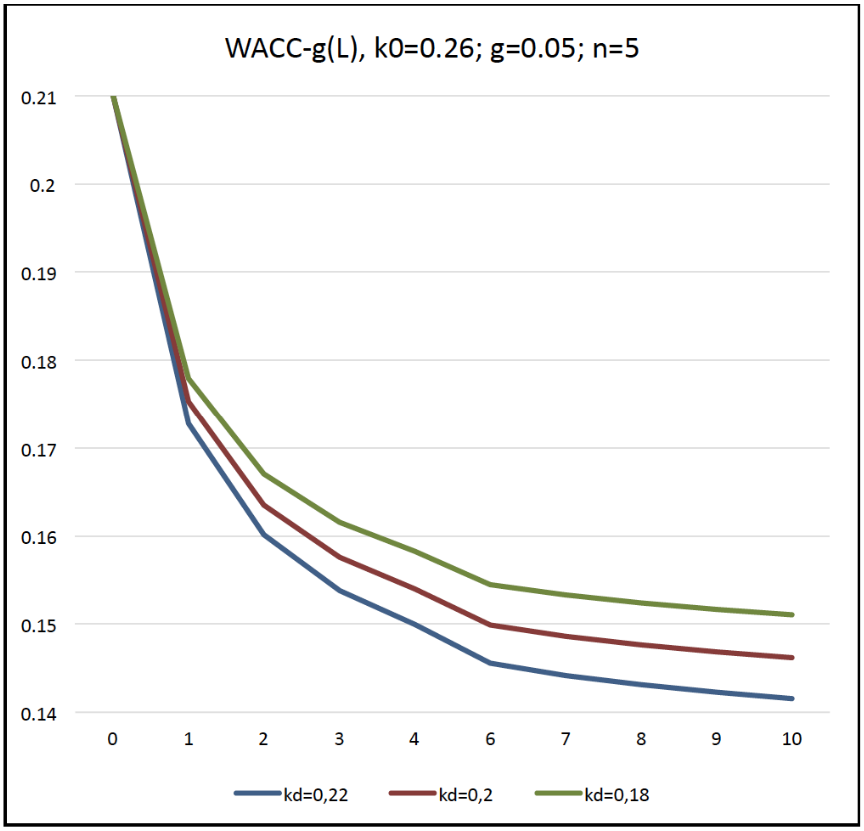

Figure 6: The dependence of discount rate, WACC–g, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; 0.2; 0.18; t=0.2; g=0.05 for five–year company.

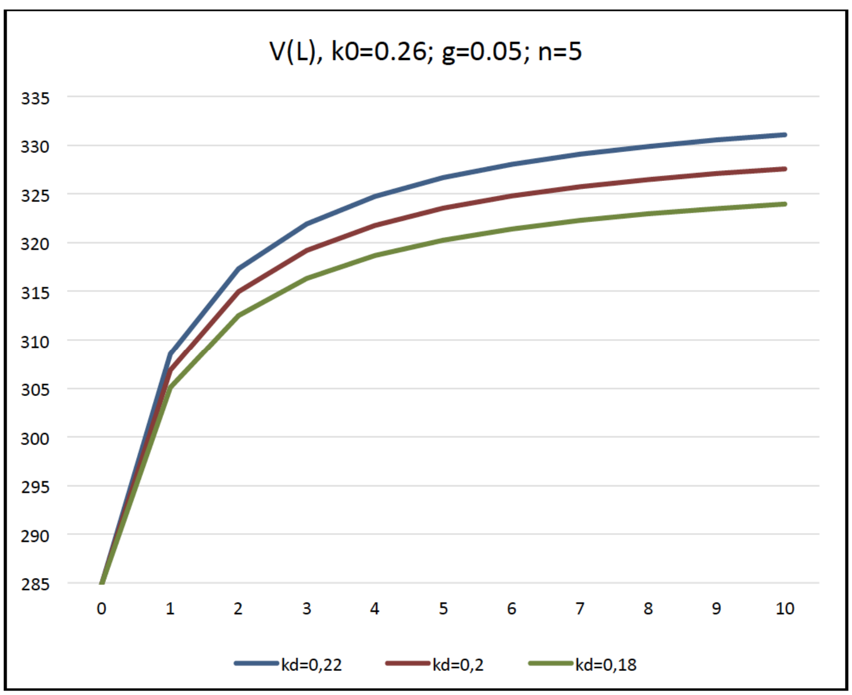

Figure 7: The dependence of company value, V, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; 0.2; 0.18; t=0.2; g=0.05 for five–year company.

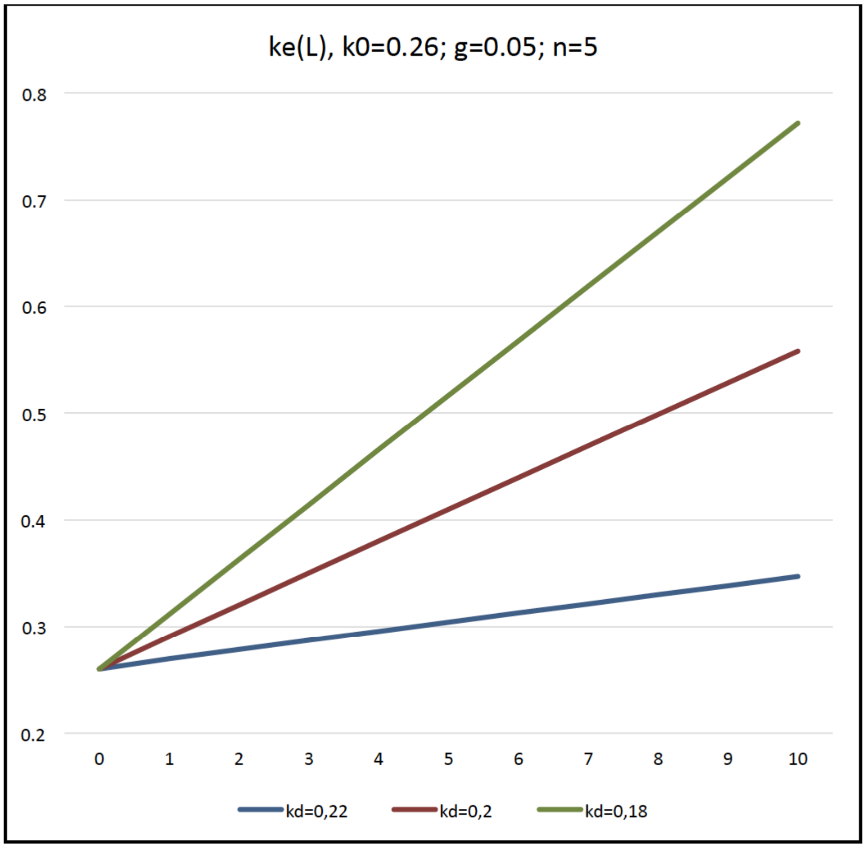

Figure 8: The dependence of equity cost, ke, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; 0.2; 0.18 t=0.2; g=0.05 for five–year company.

profits and companies with decreasing profits. And also it allows study the companies whose profits rise and fall in different periods.

3.3.1. WACC(L)

We investigate the dependence of WACC on L for companies of two ages: five and ten years old at two values of growth rates (g=0.2 and g=–0.2) (see Figures **9** and **10**). In both cases, the two WACC(L) curves start from the point (0; 0.26) and decrease with L. For positive growth rate (g=0.2) WACC is lower for five–year company, while for negative growth rate (g=–0.2) WACC is lower for ten–year company. We are seeing this effect for the first time.

3.3.2. Discount Rate WACC–g

We investigate the dependence of discount rate WACC–g on L for companies of two ages: five and ten years old at two values of growth rates (g=0.2 and g=–0.2) (see Figures **11** and **12**). In both cases, the two WACC(L) curves start from one point: (0; 0.06) for g=0.2 and (0; 0.46) for g=–0.2 and decrease with L. For positive growth rate (g=0.2) discount rate WACC–g is lower for five–year company, while for negative growth rate (g=–0.2) discount rate WACC–g is lower for ten–year company. We are seeing this effect for the first time.

3.3.3. Company Value, V

Let us study the dependence of the value of a company V on L for companies of two ages: five and ten years old at two values of growth rates (g=0.2 and g=–0.2). In both cases, the value of the company V increases with the growth of L, and the greater the age of the company corresponds to the greater value of the company V. But if, with a positive growth rate (g = 0.2), the difference in the value of V for a five–year company and a ten–year company is about 400, with negative growth rate (g=–0.2), this difference is 45 (ten times less) (see Figures **13** and **14**). So, in the competition between the age of the company and the size of the growth rate, the growth rate wins.

3.3.4. Equity Cost, ke

Let us study the dependence of the equity cost, ke, on L for companies of two ages: five and ten years old at two values of growth rates (g=0.2 and g=–0.2). In the case of a positive growth rate (g = 0.2), the cost of equity ke increases linearly with L, and the slope ke(L) for a ten–year company is greater than for a five–year

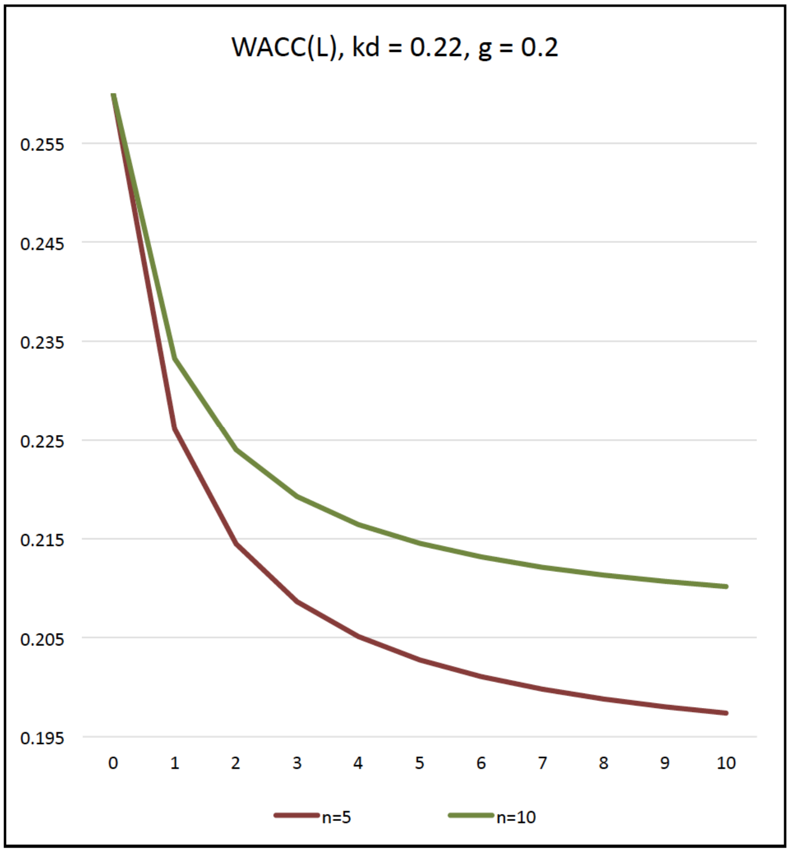

Figure 9: The dependence of WACC on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2 for five–year and ten–year companies.

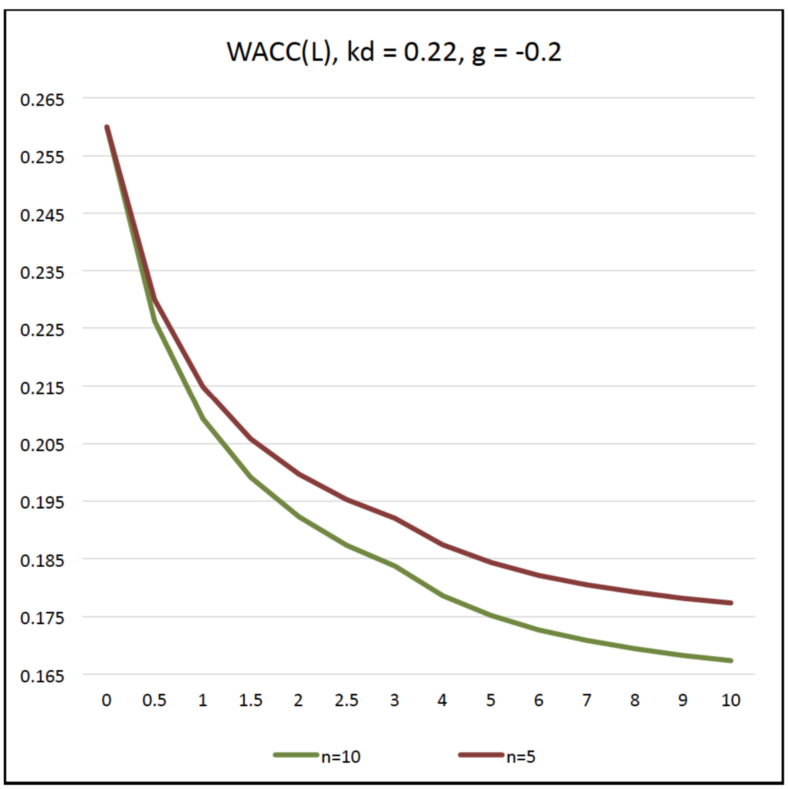

Figure 10: The dependence of WACC on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=–0.2 for five–year and ten–year companies.

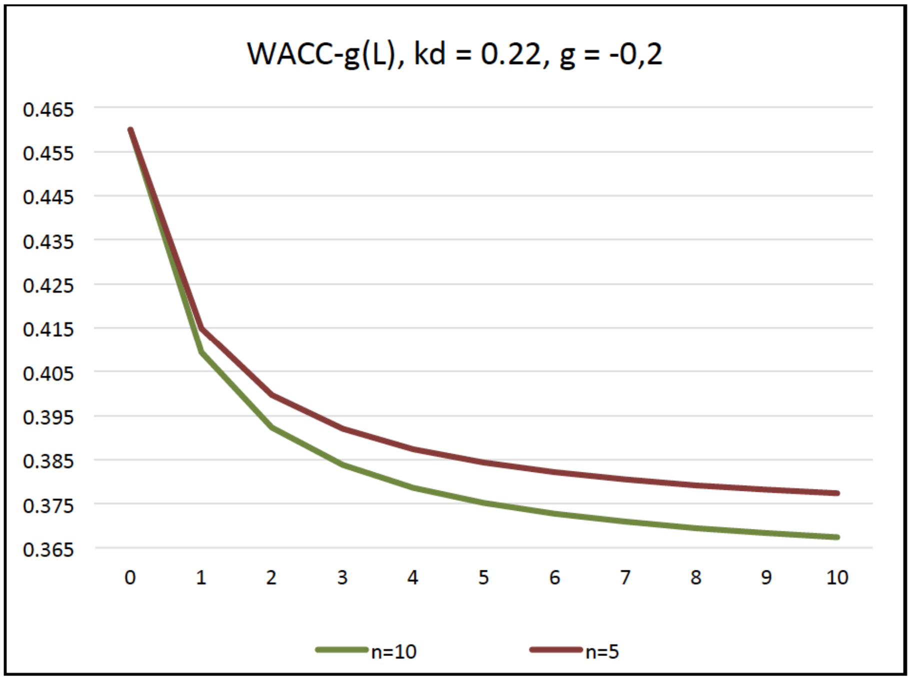

Figure 11: The dependence of discount rate, WACC–g, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=–0.2 for five–year and ten–year companies.

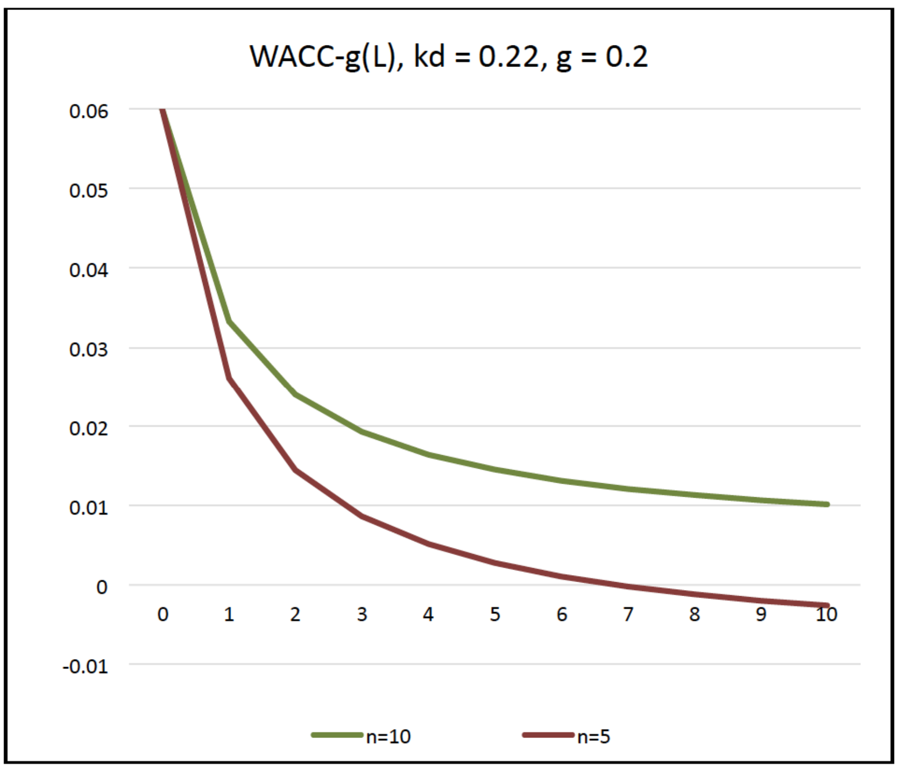

Figure 12: The dependence of discount rate, WACC–g, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2 for five–year and ten–year companies.

one. With a negative growth rate (g = –0.2), the cost of equity ke decreases linearly with increasing L, and the negative slope for a ten–year company is greater than for a five–year one.

Since the cost of equity determines the economically justified amount of dividends, this means that the dividend policy of the company when increasing profits and when decreasing profits should

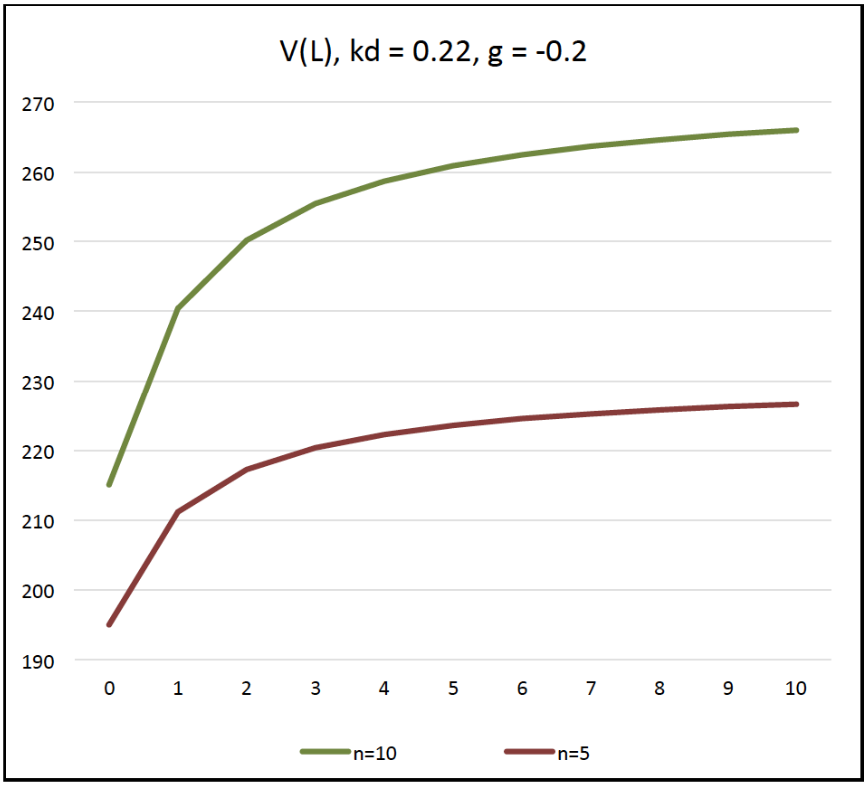

Figure 13: The dependence of company value, V, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=–0.2 for five–year and ten–year companies.

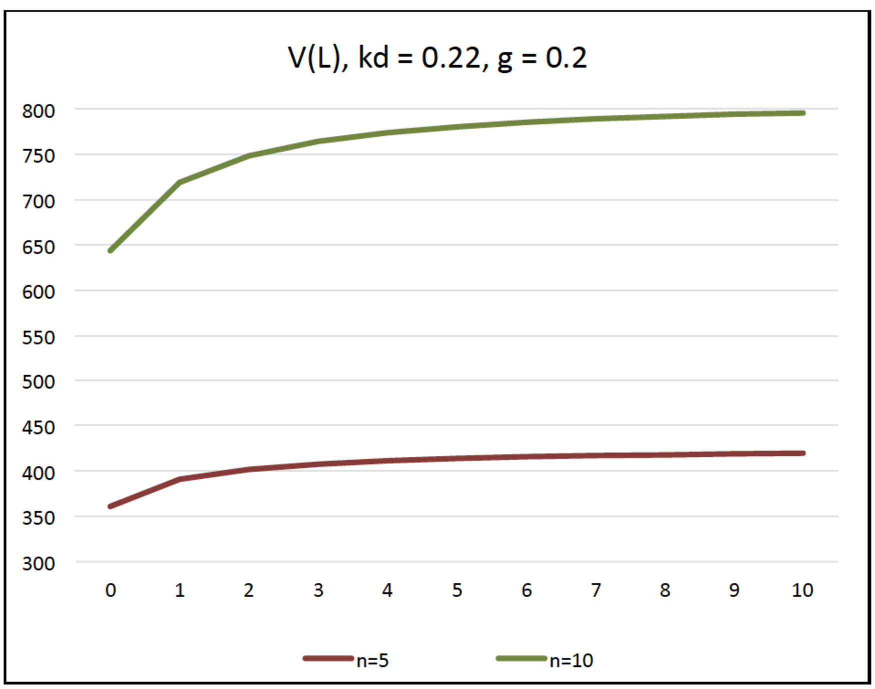

Figure 14: The dependence of company value, V, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2 for five–year and ten–year companies.

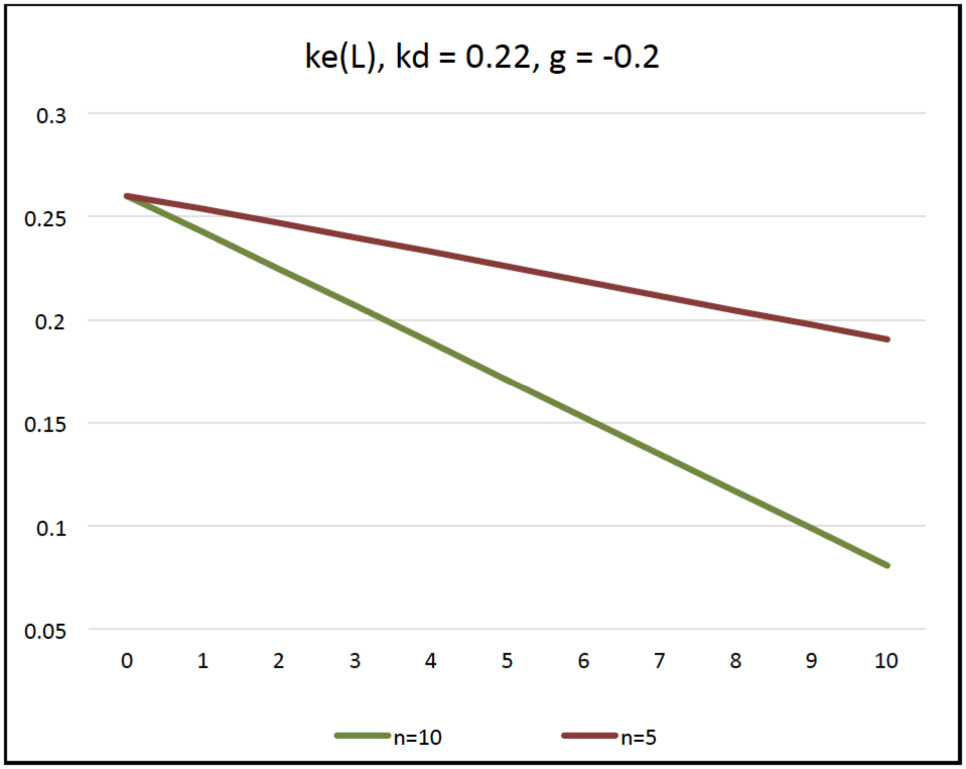

Figure 15: The dependence of equity cost, ke, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=–0.2 for five–year and ten–year companies.

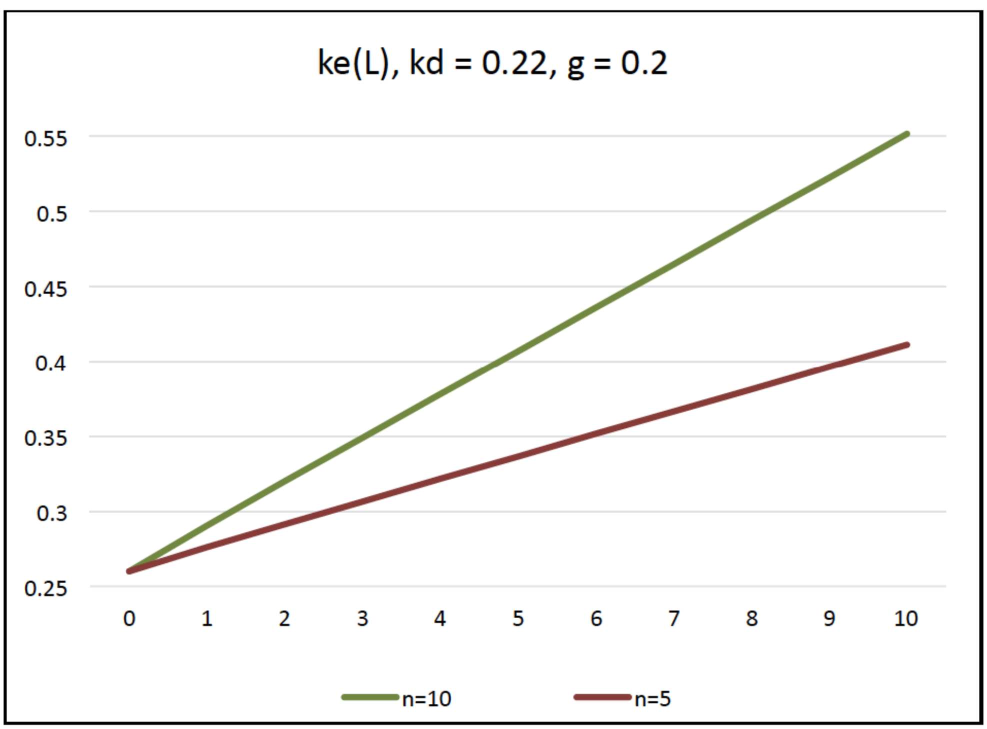

Figure 16: The dependence of equity cost, ke, on leverage level L in Generalized Brusov–Filatova–Orekhova theory (GBFO theory) with advance payments of tax on profit at k_0=0.26; k_d=0.22; t=0.2; g=0.2 for five–year and ten–year companies.

be completely different. This is a qualitatively new effect, discovered by us for the first time.

3.3.5. Results Summary

The curves (WACC–g)(L) decrease with leverage level L at all g values. The curves (WACC–g)(L) decrease with growth rate, g.

The company value V increases with leverage level L at fixed growth rate g. The company value V increases with growth rate g as well. This is a consequence of a decrease of the discount rate (WACC–g)(L) with an increase of g.

The equity cost, ke, practically linearly increases with leverage level L at all g values. The tilt angle ke(L) increases with g.

Studying the impact of the debt cost k_d on the main financial indicators, we found the following:

- all curves WACC(L) at all values of k_d start from one point (0; 0.26). WACC(L) decrease with leverage level L at all values of k_d. WACC(L) decrease with the increase of debt cost k_d. This means that tax shield advantages the decrease of the cost of raising capital.
- All curves of discount rate (WACC–g) (L) at all values of k_d start from one point (0; 0.21). (WACC–g) (L) decrease with leverage level L at all values of k_d. (WACC–g)(L) decrease with the increase of debt cost k_d. This means that the tax shield tends to lower the value of the discount rate (WACC–g) and hence increase the value of the company, V.
- All curves of company value V (L) at all values of k_d start from one point (0; 285). V (L) increases with leverage level L at all values of k_d. V(L) increases with the increase of debt cost k_d. This means that tax shield advantages the increase of the company value, V.
- All curves of equity cost, ke(L), at all values of k_d start from one point (0; 0.26) and k_e increases with leverage level L at all values of k_d. The slope of the straight line ke(L) decreases with the cost of debt kd. This means that debt cost kd impact the dividend policy of the company, because the equity cost ke determines the economically justified amount of dividends.

Studying the impact of company age, n, on main financial indicators of the company: WACC; WACC–g; V; ke, we found a huge difference between the behavior of the main financial indicators of the company with a positive and negative growth rate g. This allows you to explore companies with growing profits and companies with decreasing profits. And also it allows study the companies whose profits rise and fall in different periods.

Particular results here are as following.

We investigate the dependence of WACC on L for companies of two ages: five and ten years old at two values of growth rates (g=0.2 and g=–0.2). In both cases, the two WACC(L) curves start from the point (0; 0.26) and decrease with L. For positive growth rate (g=0.2) WACC is lower for five–year company, while for negative growth rate (g=–0.2) WACC is lower for ten–year company. We are seeing this effect for the first time.

We investigate the dependence of discount rate WACC–g on L for companies of two ages: five and ten years old at two values of growth rates (g=0.2 and g=–0.2). In both cases, the two WACC(L) curves start from one point: (0; 0.06) for g=0.2 and (0; 0.46) for g=–0.2 and decrease with L. For positive growth rate (g=0.2) discount rate WACC–g is lower for five–year company, while for negative growth rate (g=–0.2) discount rate WACC–g is lower for ten–year company. We are seeing this effect for the first time.

Let us study the dependence of the value of a company V on L for companies of two ages: five and ten years old at two values of growth rates (g=0.2 and g=–0.2). In both cases, the value of the company V increases with the growth of L, and the greater the age of the company corresponds to the greater value of the company V. But if, with a positive growth rate (g = 0.2), the difference in the value of V for a five–year company and a ten–year company is about 400, with negative growth rate (g=–0.2), this difference is 45 (ten times less). So, in the competition between the age of the company and the size of the growth rate, the growth rate wins.

We study the dependence of the equity cost, ke, on L for companies of two ages: five and ten years old at two values of growth rates (g=0.2 and g=–0.2). In the case of a positive growth rate (g = 0.2), the cost of equity ke increases linearly with L, and the slope ke(L) for a ten–year company is greater than for a five–year one. With a negative growth rate (g = –0.2), the cost of equity ke decreases linearly with increasing L, and the negative slope for a ten–year company is greater than for a five–year one.

Since the cost of equity determines the economically justified amount of dividends, this means that the dividend policy of the company when increasing profits and when decreasing profits should be completely different. This is a qualitatively new effect, discovered by us for the first time.

CONCLUSIONS

The Brusov–Filatova–Orekhova (BFO) theory has been generalized for the case of variable income and advance payments of tax on profit. The generalized Brusov–Filatova–Orekhova formula for WACC has been derived. Using this formula the dependence of WACC, discount rate, WACC–g, company capitalization, V, the equity cost, ke, on leverage level L at different values of the growth rate, g, on cost of debt capital, kd, and on company age, n has been studied. It turns out that the WACC is no longer a discount rate. The role of the discount rate is played by WACC–g, which decreases with g, while the company's value increases with g. The slope of the curve ke(L) increases with g. It turns out that at the growth rate g < g* the slope of the curve ke(L) becomes negative, which can significantly change the principles of the company's dividend policy, since the economically justified amount of dividends is equal to the cost of equity. WACC(L) as well as the discount rate, WACC–g, decrease with the increase of debt cost kd. V (L) increases with leverage level L at all values of kd. And V(L) increases with the increase of debt cost kd. This means that tax shield advantages the decrease of the cost of raising capital. Examining the behavior of the main financial parameters of the company with positive (g=0.2) and negative (g=–0.2) growth rates, we found a huge difference in their behavior. This allows you to explore companies with growing profits and companies with decreasing profits. It also allows you to study companies whose profits rise and fall in different periods.

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